Information Sciences 450 (2018) 141–156

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Quaternion polar harmonic Fourier moments for color images

Chunpeng Wang[a,c], Xingyuan Wang[b,c,*], Yongwei Li[d], Zhiqiu Xia[c,*], Chuan Zhang[c]

[a] School of Information, Qilu University of Technology (Shandong Academy of Sciences), Jinan, 250353, China
[b] School of Information Science and Technology, Dalian Maritime University, Dalian, 116026, China
[c] Faculty of Electronic Information and Electrical Engineering, Dalian University of Technology, Dalian, 116023, China
[d] Department of Information and Communication Systems (IKS), Mid Sweden University, Sweden

ARTICLE INFO

Article history:
Received 15 November 2016
Revised 14 March 2018
Accepted 14 March 2018
Available online 16 March 2018

Keywords:
Quaternion polar harmonic Fourier moments
Orthogonal moment
Moment invariant
Image reconstruction
Phase
Object recognition

ABSTRACT

This paper proposes quaternion polar harmonic Fourier moments (QPHFM) for color image processing and analyzes the properties of QPHFM. After extending Chebyshev–Fourier moments (CHFM) to quaternion Chebyshev-Fourier moments (QCHFM), comparison experiments, including image reconstruction and color image object recognition, on the performance of QPHFM and quaternion Zernike moments (QZM), quaternion pseudo-Zernike moments (QPZM), quaternion orthogonal Fourier-Mellin moments (QOFMM), QCHFM, and quaternion radial harmonic Fourier moments (QRHFM) are carried out. Experimental results show QPHFM can achieve an ideal performance in image reconstruction and invariant object recognition in noise-free and noisy conditions. In addition, this paper discusses the importance of phase information of quaternion orthogonal moments in image reconstruction.

1. Introduction

Moments are used to characterize the distribution of random variables in statistics and the spatial distribution of substances in mechanics. Two-dimensional density distribution functions can be obtained from binary or grayscale images. Moment theory can be applied to such distribution functions to describe the images. In 1962, Hu [14] used geometric moments and moment invariants as the features of multidistortion invariant images to perform image recognition. Complex [1] and rotational moments [2] are distortion invariants, too. However, geometric, complex, and rotational moments suffer from information redundancy, resulting in images that are difficult to reconstruct. To overcome this shortcoming, Teague [27] proposed using continuous orthogonal image moments—Legendre moments (LM) and Zernike moments (ZM)—in describing images. Orthogonal image moments can be used to reconstruct images, and orthogonal invariants are robust to rotation, scaling, and translation. Orthogonal moments can be divided into 2 groups, namely continuous orthogonal moments, which are based on continuous orthogonal polynomials, and discrete orthogonal moments, which are based on discrete orthogonal polynomials. Continuous orthogonal moments use a continuous function as a kernel function, and the inner product between image and kernel function is carried out in a continuous space; such moments include pseudo-Zernike moments (PZM) [28], orthogonal Fourier-Mellin moments (OFMM) [24], Chebyshev–Fourier moments (CHFM) [20], radial harmonic Fourier moments (RHFM) [21], Bessel-Fourier moments (BFM) [35], polar harmonic transform (PHT) [36], and ex-

* Corresponding author.
E-mail addresses: wangxy@dlut.edu.cn (X. Wang), xzqjsdtc@163.com (Z. Xia).

https://doi.org/10.1016/j.ins.2018.03.040
0020-0255/

ponential moments (EM) [19]. However, discrete orthogonal moments use a discrete function as a kernel function, and the inner product between image and kernel function is carried out in a discrete space (e.g., Tchebichef moments (TM) [15], Krawtchouk moments (KM) [37], Racah moments (RM) [38], dual Hahn moments (DHM) [39]). Continuous orthogonal moments are robust to translation, scaling, and rotation, and they can be applied to pattern recognition, digital watermarking, image retrieval, and so on. Discrete orthogonal moments do not require the process of coordinate transformation and approximation of the integral, which avoids the deviation caused by calculation and significantly extends their application in image analysis (e.g., image compression, edge detection, 2D reconstruction, image registration, digital watermarking).

Although much developed in recent years, the theory of image moments mainly focuses on grayscale images, yet with the continuous improvement of computer performance, color images are now able to provide much more abundant information compared with grayscale images, so color images are drawing more and more attention from researchers in the related fields. Current research on color image moments is mostly based on the intensity or a single channel within a color space of color images, which discards the information and relationship between color components within a specific color space.

Recently, quaternion-based image representation and application have become increasingly important in the field of image processing, and some of the quaternion-based methods are already applied in several domains related to image processing (e.g., quaternion Fourier transform [7, 32], quaternion wavelet transform [4, 8], quaternion singular value decomposition [9, 17]). Quaternion-based image processing regards a color pixel as a vector, which holds the relationship between different channels of a color image. Based on this theory, Guo et al. [11] extended traditional Fourier-Mellin moments into a quaternion form (i.e., quaternion Fourier-Mellin moments (QOFMM)) and successfully applied it to color image registration using the constructed geometric invariant features. Subsequently, Chen et al. [6] proposed quaternion Zernike moments (QZM), built a complete feature-invariant set, and achieved an outstanding result within color object recognition and color image registration. Guo et al. [10] proposed novel quaternion moment descriptors for color images in the Cartesian coordinates, which can reduce computational complexity and improve numerical stability in the field of color image processing and object recognition. Shao et al. [23] proposed quaternion Bessel-Fourier moments (QBFM) and analyzed the importance of the phase information contained in quaternion orthogonal moments from the perspective of color image reconstruction. Chen et al. [5] summarized quaternion-type moments, including quaternion rotational moments (QROTM), QOFMM, QZM, and quaternion pseudo-Zernike moments (QPZM), and evaluated their performance of in the field of color image reconstruction, face recognition, and image registration. Additionally, they discussed the selection of quaternions in the calculation process. They also defined geometric invariant feature descriptors according to the derived rotation angle estimation algorithm and applied them in color object recognition. In recent years, some of the traditional nonorthogonal and orthogonal moments have been developed into the form of quaternion moments, like quaternion exponent moments (QEM) [31], quaternion radial Tchebichef moments (QRTM) [29], quaternion Legendre-Fourier moments (QLFM) [3], quaternion radial harmonic Fourier moments (QRHFM) [33], and so on.

Although researchers have proposed a variety of quaternion image orthogonal moments, most of them have shown poor performance in image reconstruction. In this paper, a new quaternion image orthogonal moment, namely quaternion polar harmonic Fourier moments (QPHFM), is proposed. The properties, including its invariance and relationship to and distinction from complex PHFM, are further analyzed. After proposing the quaternion form of CHFM (i.e., QCHFM) comparison experiments are conducted on the performance in image reconstruction and color object recognition of QPHFM and QOFMM, QZM, QPZM, QRHFM, and QCHFM. Experimental results show that QPHFM has the best image reconstruction performance and performs superbly in invariant object recognition in noise-free and noisy conditions.

This paper is organized as follows. Section 2 introduces the quaternion algebra and PHFM. In Section 3, the definition and properties of QPHFM are discussed in detail. Section 4 presents the comparative experiments of QPHFM with QOFMM, QZM, QPZM, QRHFM, and QCHFM in terms of image reconstruction and object recognition. Section 5 concludes.

2. Preliminaries

2.1. Quaternion algebra

Quaternions were first proposed by British mathematician Hamilton in 1843 [12]. They can be interpreted as generalizations of complex numbers. The formula of a quaternion is as follows:

$$q = a + b\mathrm{i} + c\mathrm{j} + d\mathrm{k}, \tag{1}$$

where a is the real part of the quaternion q; $b\mathrm{i} + c\mathrm{j} + d\mathrm{k}$ is the imaginary part of the quaternion q; a, b, c, and d are real numbers; and i, j, and k are complex operators that satisfy the following conditions:

$$\begin{array}{l} \mathrm{i}^2 = \mathrm{j}^2 = \mathrm{k}^2 = \mathrm{ijk} = -1 \\ \mathrm{ij} = \mathrm{k}, \mathrm{jk} = \mathrm{i}, \mathrm{ki} = \mathrm{j} \\ \mathrm{ji} = -\mathrm{k}, \mathrm{kj} = -\mathrm{i}, \mathrm{ik} = -\mathrm{j} \end{array}. \tag{2}$$

The conjugate and the magnitude of the quaternion q are $\bar{q} = a - b\mathrm{i} - c\mathrm{j} - d\mathrm{k}$ and $|q| = \sqrt{a^2 + b^2 + c^2 + d^2}$, respectively.

A quaternion can be considered the combination of a scalar part and a vector part: $q = s(q) + v(q)$, where $s(q) = a$ and $v(q) = b\mathrm{i} + c\mathrm{j} + d\mathrm{k}$. A quaternion with $s(q) = 0$ can be also referred to as a pure quaternion, and a quaternion with unit magnitude is called a unit quaternion.