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Adaptive tabu search with strategic oscillation for the bipartite boolean quadratic programming problem with partitioned variables

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ABSTRACT

The bipartite boolean quadratic programming problem with partitioned variables (BQP-PV) is an NP-hard combinatorial optimization problem that accommodates a variety of real-life applications. We propose an adaptive tabu search with strategic oscillation (ATS-SO) approach for BQP-PV, which employs a multi-pass search framework where each pass consists of an initial constructive phase, an adaptive tabu search phase and a frequency-driven strategic oscillation phase. In particular, the adaptive tabu search phase combines different move operators to collectively conduct neighborhood exploration and an adaptive tabu tenure management mechanism that obviates the task of determining a proper tabu tenure. The frequency-driven strategic oscillation phase diversifies the search when the search reaches a critical solution, drawing on a destructive procedure to unassign some variables by reference to frequency memory and a constructive procedure to re-assign these variables utilizing both frequency memory and problem specific knowledge. Computational experiments on five classes of problem instances indicate that the proposed ATS-SO algorithm is able to find improved solutions for 14 instances and match the best known solutions for all remaining instances, whereas no previous method has succeeded in finding the previous best solutions for all instances. Statistical tests indicate that ATS-SO significantly outperforms the state-of-the-art algorithms in the literature.

1. Introduction

Let $S_1, S_2, \dots, S_p$ be a partition of a set $I = \{1, 2, \dots, m\}$ and let $T_1, T_2, \dots, T_k$ be a partition of a set $J = \{1, 2, \dots, n\}$. A profit q_{ij} is associated with each $(i, j) \in I \times J$. Further, profits c_i and d_j are associated with each $i \in I$ and each $j \in J$ and we let c_0 denote a constant. Then the bipartite boolean quadratic programming problem with partitioned variables (BQP-PV) [32] is

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to

$$\text{Maximize} \sum_{i=1}^{m}\sum_{j=1}^{n} q_{ij}x_i y_j + \sum_{i=1}^{m} c_i x_i + \sum_{j=1}^{n} d_j y_j + c_0 \tag{1}$$

Subject to

$$\sum_{i\in S_r} x_i = 1, \text{ for } r = 1, 2, \ldots, p \tag{2}$$

$$\sum_{j\in T_u} y_j = 1, \text{ for } u = 1, 2, \ldots, k \tag{3}$$

$$x_i, y_j \in \{0, 1\}, \text{ for } i \in I, j \in J. \tag{4}$$

A graph theoretic interpretation of BQP-PV can be given by letting $G = (I, J, E)$ be a bipartite graph and $S_1, S_2, \ldots, S_p$, and $T_1, T_2, \ldots, T_k$ be partitions of the node sets I and J respectively. A subgraph $G' = (I', J', E')$ is said to be a *representative subgraph* of G if I' contains exactly one node from each S_r, $r = 1, 2, \ldots, p$ and J' contains exactly one node from each T_u, $u = 1, 2 \ldots, k$. Given costs c_i and d_j for each $i \in I$ and $j \in J$, and a weight q_{ij} for each edge $(i, j) \in E$, the BQP-PV problem is equivalent to the *maximum weight representative subgraph problem* on G, which finds a representative subgraph of G to maximize the sum of the edge and node weights.

BQP-PV is closely related to the well-known quadratic assignment problem (QAP) [6] and quadratic semi-assignment problem (QSAP) [10,11]. Details of how these problems are interrelated can be found in [32]. Because of the unifying nature of BQP-PV, the problem is NP-hard and applications of QAP and QSAP can be viewed as potential applications of BQP-PV [32]. BQP-PV also includes applications in clustering and bioinformatics [36,38] and robust optimization [2,43], as well as applications involving scenarios for maximizing collaborative output, such as selection of negotiating teams, selection of component manufacturers and assembly units, etc. BQP-PV belongs to the class of combinatorial optimization problems with interaction costs [25].

BQP-PV can be reformulated as a Boolean Bipartite Quadratic Programming (BBQP) problem [12,18,20,30,31] by introducing large penalties for violating constraints (18) and (19) into the objective function using ideas similar to those discussed in [3,22,23]. Hence, algorithms designed for the BBQP problem can also be used for solving the reformulated BQP-PV problem instances. There exist many methods in the literature for handling BBQP problem instances, including a branch-and-bound approach [12], constructive and local search heuristics [20], tabu search and hybrid metaheuristic [18] and a multi-component approach [21]. However, penalty factors imposed in the reformulation limit the computational flexibility to exploit the special structure of BQP-PV, which is essential to obtain high-performance algorithms.

The BQP-PV model is introduced in [32] and local search, tabu search, and variable neighborhood search algorithms using swap, concurrent swap, and optimized swap move operators independently are investigated. Moreover, the compound effect of various combinations of these move operators are also investigated. Extensive experiments reveal that the hybrid algorithms outperform the algorithms that use a single move operator and using tabu search is able to significantly improve performance compared to using its local search counterparts. Further computational comparisons disclose that these heuristic algorithms produce much better solutions than the general solver CPLEX 12.5 while consuming much less running time.

Theoretical properties of BQP-PV are studied by Custic and Punnen [8] who establish a closed-form expression for computing the average value of the objective function along with various domination analysis results.

In this paper, we extended the work in [32] to propose an adaptive tabu search with strategic oscillation (ATS-SO) approach for handling difficult BQP-PV instances. The proposed algorithm is essentially a multi-pass procedure. Each pass starts with a randomized greedy constructive phase to generate a starting solution. Then an adaptive tabu search phase for intensification and a strategic oscillation phase for diversification are alternatively performed to ensure an effective examination of the search space where the starting solution lies in. The execution of each pass is terminated when the number of performed iterations surpasses the prescribed maximum number of iterations.

Our proposed ATS-SO approach includes the following original features. First, we develop new swap move operators and combine them with probabilistic information to efficiently conduct neighborhood exploration. Then, we apply the move operators within the tabu search framework, where an adaptive tabu tenure management strategy exploits information from the search history to automatically adjust tabu tenure. Finally, we employ a strategic oscillation scheme to diversify the search once the search reaches a critical solution (identified as the solution in the last iteration of tabu search).

The rest of the paper is organized as follows. Section 2 introduces previous research work related to our proposed algorithm. Section 3 describes the proposed adaptive tabu search approach incorporating strategic oscillation. Section 4 provides preliminary experiments to evaluate the merit of each component in the proposed algorithms. Section 5 reports experimental results and comparisons with state-of-the-art algorithms from the literature. Concluding remarks are given in Section 6.