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Generalized exponential autoregressive models for nonlinear time series: Stationarity, estimation and applications

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ABSTRACT

The generalized exponential autoregressive (GExpAR) models are extensions of the classic exponential autoregressive (ExpAR) model with much more flexibility. In this paper, we first review some development of the ExpAR models, and then discuss the stationary conditions of the GExpAR model. A new estimation algorithm based on the variable projection method is proposed for the GExpAR models. Finally, the models are applied to two real-world time series modeling and prediction. Comparison results show that (i) the proposed estimation approach is much more efficient than the classic method, (ii) the GExpAR models are more powerful in modeling the nonlinear time series.

1. Introduction

Many dynamic phenomena such as brain wave records, car vibrations, animal populations and electric circuits can be regarded as stochastic processes. In the early stage, these stochastic processes are often approximated by linear time series models, e.g., the autoregressive (AR) model, the autoregressive moving average (ARMA) model [16]. Linear models provide us convenient tools for controlling and forecasting.

However, some phenomena essentially display nonlinear behaviors, such as time irreversibility, asymmetry, and self-sustained stochastic cyclical behavior, which cannot be explained well by linear time series models. Realizing this, researchers have proposed numerous nonlinear time series models since the late 1970s. The ExpAR model is one of the most excellent nonlinear models in the early stage. It was first used to model the ship rolling data by Ozaki and Oda [38]. Later, in a series of works [19,27,28], the ExpAR model was used to explain the phenomena like jump phenomena, amplitude dependent frequency shifts and perturbed limit cycles. Given a time series $\{x_1, x_2, x_3, \ldots\}$, an ExpAR model of order p is defined as

$$x_t = \{c_1 + \pi_1 e^{-\gamma \cdot x_{t-1}^2}\} \cdot x_{t-1} + \cdots + \{c_p + \pi_p e^{-\gamma \cdot x_{t-1}^2}\} \cdot x_{t-p} + \varepsilon_t \tag{1}$$

where ε_t is an i.i.d random variable and independent with x_i, and c_i, π_i, γ are the unknown parameters which need to be estimated from observations.

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Model (1) is the classic form of the ExpAR model. Several variants were proposed later. In [29], Ozaki gave a more complex form of the ExpAR model where the autoregressive coefficients are Hermite type polynomials:

$$x_t = \sum_{i=1}^{p} \left\{ c_i + \left(\pi_0^{(i)} + \sum_{j=1}^{k_i} \pi_j^{(i)} x_{t-1}^j \right) \cdot e^{-\gamma \cdot x_{t-1}^2} \right\} \cdot x_{t-i} + \varepsilon_t \tag{2}$$

where $\pi_j^{(i)}, c_i, \gamma$ are constants which need to be estimated.

In [20], Tong considered a modified ExpAR model, which has been widely used in financial data. It is defined as

$$x_t = \sum_{i=1}^{p} \{c_i + \pi_i \cdot e^{-\gamma \cdot x_{t-i}^2}\} \cdot x_{t-i} + \varepsilon_t. \tag{3}$$

Another extended version of the ExpAR was suggested by Teräsvirta in [36]:

$$x_t = (c_0 + \pi_0 e^{-\gamma (x_{t-d} - z)^2}) + \sum_{j=1}^{p} \{c_j + \pi_j e^{-\gamma (x_{t-d} - z)^2}\} \cdot x_{t-j} + \varepsilon_t \tag{4}$$

where z is a scalar parameter and d is an integer number.

Engle and Bollerslev introduced the Autoregressive Conditional Heteroskedasticity (ARCH) and the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models in [5,13], which were regarded as the most characteristic examples of nonlinear models in conditional variance. LeBaron [25] first proposed the ExpAR model with GARCH error. Later, Koutmos generalized LeBaron's model in [24] and used it to study the daily stock returns in some equity markets of the Pacific Basin area. In [23], P.Katsiampa suggested the ExpAR-ARCH and ExpAR-GARCH models, combining the ExpAR model for the conditional mean and the ARCH or GARCH model for the conditional variance. Different from LeBaron and Koutmos, P.Katsiampa's model contains the lag of the variable in the exponential term.

In real-world applications, the stationarity is usually an important precondition for efficient use of time series models. In fact, the initial idea of the ExpAR model [38] is to keep the model stationary. Although Ozaki [27,29,30] gave some sufficient conditions for the ExpAR model, they are somewhat restrictive. In this paper, we will give less restrictive conditions for the ExpAR models, and discuss the ergodic conditions for the GExpAR models.

For the parameter estimation of the ExpAR model, it is essentially a nonlinear optimization problem. However, Haggan and Ozaki [19] converted this nonlinear optimization problem into a linear least squares problem by fixing the parameter γ at one of a grid values. This method will work if there is only one nonlinear parameter in the ExpAR model. For the GExpAR model, there may be several nonlinear parameters. In this case, if we select the nonlinear parameters from a high-dimensional grid of values, the computational load will grow heavily (as will be seen in Section 4). Fortunately, the ExpAR model and GExpAR models have the form of linear combination of nonlinear functions, i.e., some of the parameters appear linearly. Parameter estimation for this kind of models belongs to a class of separable nonlinear least squares (SNLLS) problems. The variable projection (VP) method proposed by Golub and Pereyra [17,18] is a high efficient algorithm for solving SNLLS problems. This method eliminates the linear parameters from the problem, and thus reduces the dimension of the parameter space and resulting in a better-conditioned problem. In this paper, we adopt the VP approach to estimate the parameters of the ExpAR and GExpAR models.

The remainder of this paper is organized as follows. Section 2 explores the geometric ergodicity of the GExpAR models. Section 3 shows the implementation of the VP method to optimize the GExpAR model. Numerical results and comparison on two sets of real world data are provided in Section 4. Finally, we summerize the conclusion of this paper in Section 5.

2. The ergodicity of the GExpAR model

In this section, we discuss the stability conditions of the GExpAR models. For clarity, we first introduce some variables and symbols. Let us define:

$$\boldsymbol{X}_t = (x_t, \ldots, x_{t-p+1})^T,$$

$$\boldsymbol{\varepsilon}_t = (\varepsilon_t, 0, \ldots, 0)^T,$$

$$\boldsymbol{C}_0 = (c_0, 0, \ldots, 0)^T,$$

$$\boldsymbol{A}(\boldsymbol{X}) = \begin{pmatrix} \varphi_1(\boldsymbol{X}) & \varphi_2(\boldsymbol{X}) & \cdots & \varphi_{p-1}(\boldsymbol{X}) & \varphi_p(\boldsymbol{X}) \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix}.$$