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Method for three-way decisions using ideal TOPSIS solutions at Pythagorean fuzzy information

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ABSTRACT

Pythagorean fuzzy sets (PFSs) as a new generalization of intuitionistic fuzzy sets (IFSs) can effectively handle uncertain information more flexibly in the process of decision making. As a natural extension of three-way decisions with decision-theoretic rough sets (DTRSs), this paper proposes a new model of three-way decisions and develops the corresponding decision-making procedure based on Pythagorean fuzzy information systems. With respect to the results reported in most of the existing papers, we consider a general situation that the information system does not have the class label. In this case, we can encounter two challenges and need to reinterpret the loss function and the conditional probability. Considering the properties of PFSs, we firstly introduce the Pythagorean fuzzy number (PFN) into DTRSs, which can provide a new interpretation for the loss function. Then, we construct a new model of Pythagorean fuzzy decision-theoretic rough sets (PFDTRSs) based on the Bayesian decision procedure. With respect to the conditional probability, we effectively utilize the technique for order preference by similarity to ideal solution (TOPSIS) method to estimate it. Furthermore, we design a decision-making procedure of three-way decisions-based ideal solutions in the Pythagorean fuzzy information system. Our proposed method not only takes the decision risk into consideration, but also tells us how to choose the action for each project and gives its corresponding semantic explanation, which can replenish the decision results of TOPSIS. Finally, we expound the application of three-way decisions by an example of the research and development (R&D) project selection and validate our method via the comparison analysis.

1. Introduction

Three-way decisions are to divide a universal set into three pair-wise disjoint regions by developing an appropriate strategy [43]. According to some different contexts, it can be interpreted by the corresponding semantics, such as three-way decisions with concept analysis [44], sequential three-way decisions [14,15], three-way decisions with conflict analysis [11], three-way decisions with granular computing [42], three-way decisions with cognitive computing [24,43] and three-way decisions with decision-theoretic rough sets (DTRSs) [37,38]. Among them, three-way decisions with DTRSs proposed by Yao et al. [37] and Yao and Wong [38] are an important research direction. It has an acceptance decision, a deferment

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decision and a rejection decision [27,39–41]. It has attracted the attention of many researchers. For example, in order to compute the thresholds, Azam and Yao [2] designed a game-theoretic method. Hu [8] constructed three-way decisions space. Three-way decisions with DTRSs also have been applied in many fields, e.g., the information filtering [12], the investment decision-making [16,20], the text classification [13], the cluster analysis [45], the government decision-making [21], the risk decision-making [18], the web-based support systems [36], etc. The loss function and the conditional probability constitute two key elements of DTRSs. At present, fuzzy evaluation scenarios for three-way decisions provide a new semantics and push its research [16,17].

Recently, Pythagorean fuzzy sets (PFSs) proposed by Yager [33,35] extend the application of fuzzy sets [46]. It inherits the duality property of intuitionistic fuzzy sets (IFSs) [1,31,32] and depicts an evaluation by two elements, i.e., a membership degree and a non-membership degree [34]. However, the square sum of the two elements of PFSs is equal to or less than 1. As the example presented in Ref. [35], the membership degree and the non-membership degree of one alternative are $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$, respectively. In this case, $\frac{\sqrt{3}}{2}+\frac{1}{2}\geq 1$ and $(\frac{\sqrt{3}}{2})^2+(\frac{1}{2})^2\leq 1$ hold [47]. It implies that IFSs cannot describe this case. Fortunately, PFSs generalize the concept of IFSs [47] and have the corresponding operational laws, which has successfully applied to some complex practical decision-making situations, e.g., the investment decision-making [25], the candidate selection of Asian Infrastructure Investment Bank [28] and the service quality of domestic airline [47]. Dick et al. [4] examined the lattice-theoretic properties of PFSs, and then extended them to the unit disc of the complex plane. Combining Einstein operations, Garg [7] analyzed some generalized Pythagorean fuzzy aggregations. Peng and Yang [25] analyzed the subtraction and division operations and discussed their properties in detail. Besides, Peng and Yang [26] also proposed Pythagorean fuzzy Choquet integral aggregation operators. Considering the decision maker's psychological behavior, Ren et al. [28] introduced the TODIM approach to the Pythagorean fuzzy decision environment. Considering the hierarchical problems, Zhang [48] constructed Pythagorean fuzzy QUALIFLEX method by using a new closeness index.

With the aforementioned literature, PFSs provide us a novel evaluation format to measure the fuzzy environment, especially when we utilize the positive and negative sides to depict a question [1,18]. Based on the Pythagorean fuzzy environment, we introduce PFSs into DTRSs and develop a novel method for deriving three-way decisions. In our real life, we can encounter that the information system does not have the class label or the decision attribute. For example, in the multi-criteria decision-making, some research works only involve conditional attributes, see Refs. [28,48]. In this situation, we encounter two challenges: (1) How to explain the loss function and construct the basic model of three-way decisions; (2) how to estimate the conditional probability. On these two issues, we firstly discuss the new interpretation of the loss function with PFN. In the framework of Bayesian decision procedure, we further construct the basic model of Pythagorean fuzzy decision-theoretic rough sets (PFDTRSs). With respect to the conditional probability, we need to design a new approach to calculate it in the Pythagorean fuzzy information systems without the class label. Fortunately, the technique for order preference by similarity to ideal solution (TOPSIS) method provides us a novel viewpoint for estimating the conditional probability, which is measured by the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution [9,47]. At this moment, two reference points, i.e., the positive ideal solution and the negative ideal solution, just correspond to the class information. According to this idea, we successfully adopt the TOPSIS method to estimate the conditional probability and propose three-way decisions based ideal solutions. The main contributions of our study are summarized below: (1) This paper gives a novel interpretation of loss functions and expands the scope of applications of three-way decisions; (2) this paper fully integrates the methods of TOPSIS and three-way decisions. On the one hand, the TOPSIS method can estimate the conditional probability for three-way decisions. On the other hand, three-way decisions take the decision risk into consideration and availably judge the actions of different alternatives of TOPSIS, e.g., under the minimum risk, our proposed method can advise which alternatives need to be accepted and which alternatives should be deferred.

The remainder of the paper is organized as follows: Some basic concepts of PFSs are provided in Section 2. Under the Pythagorean fuzzy environment, a basic model of PFDTRSs is constructed in Section 3. The ideal solutions for three-way decisions with PFDTRSs is developed in Section 4. Section 5 uses an example to elaborate our proposed method. We further design the comparison analysis in Section 6. Section 7 concludes our work and indicates future studies.

2. Pythagorean fuzzy sets (PFSs)

PFSs [33,34] extend the concepts of FSs [46] and IFSs [1]. In what follows, some basic concepts of PFSs are briefly reviewed in this section [33,34,47,48].

Definition 1. [47] Let S be a fixed set. A Pythagorean fuzzy set (PFS) P on S can be represented as the following mathematical symbol:

$$P=\{<s, P(\mu_P(s), \nu_P(s))> | s\in S\}, \tag{1}$$

where the functions $\mu_P(s)$: $S\to[0, 1]$ and $\nu_P(s)$: $S\to[0, 1]$ denote a membership degree and a non-membership degree of s to P, respectively. For every $s\in S$, it satisfies the condition: $0\leq(\mu_P(s))^2+(\nu_P(s))^2\leq 1$. The degree of indeterminacy of s to P is $\pi_P(s)=\sqrt{1-(\mu_P(s))^2-(\nu_P(s))^2}$. For simplicity, Zhang and Xu [47] called $P(\mu_P(s), \nu_P(s))$ as a Pythagorean fuzzy number (PFN), denoted by $\beta=P(\mu_\beta, \nu_\beta)$, where μ_β, $\nu_\beta\in[0, 1]$, $\pi_\beta=\sqrt{1-(\mu_\beta)^2-(\nu_\beta)^2}$ and $(\mu_\beta)^2+(\nu_\beta)^2\leq 1$.