

# Accepted Manuscript

Material Implications in Lattice Effect Algebras

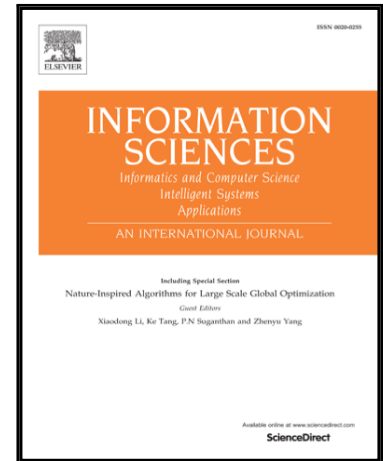
R.A. Borzooei, A. Dvurečenskij, A.H. Sharafi

PII: S0020-0255(17)31170-2  
DOI: [10.1016/j.ins.2017.12.049](https://doi.org/10.1016/j.ins.2017.12.049)  
Reference: INS 13343

To appear in: *Information Sciences*

Received date: 1 April 2017  
Revised date: 16 August 2017  
Accepted date: 25 December 2017

Please cite this article as: R.A. Borzooei, A. Dvurečenskij, A.H. Sharafi, Material Implications in Lattice Effect Algebras, *Information Sciences* (2017), doi: [10.1016/j.ins.2017.12.049](https://doi.org/10.1016/j.ins.2017.12.049)



This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

# Material Implications in Lattice Effect Algebras

R. A. Borzooei<sup>(a)</sup>, A. Dvurečenskij<sup>(b,c)</sup>, A. H. Sharafi<sup>(a)</sup>,

<sup>(a)</sup>*Department of Mathematics, Shahid Beheshti University, Tehran, Iran*  
 borzooei@sbu.ac.ir, am.hsn.sharafi@gmail.com

<sup>(b)</sup>*Mathematical Institute, Slovak Academy of Sciences, Bratislava, Slovakia*

<sup>(c)</sup>*Department of Algebra Geom., Palacký University, Olomouc, Czech Republic*  
 dvurecen@mat.savba.sk

## Abstract

In this paper we show that it is not acceptable using the minimal conditions of implication introduced for orthomodular lattices as minimal conditions of a good implication for lattice effect algebras. Hence, we first define the concept of a partial t-norm on bounded lattices. Then we obtain the concept of a pt-implication on bounded involutive lattices by introducing some conditions using partial t-norms. Furthermore, we conclude that the Sasaki arrow which is a very important implication in lattice effect algebras is the best pt-implication on them. Finally, applying the Sasaki arrow, we construct a fuzzy implication on a lattice effect algebra and also we prove that it is a weak pt-implication as well.

**Keywords:** Lattice effect algebra, pt-implication, Sasaki arrow, fuzzy implication, partial t-norm.

1

## 1 Introduction

Since the class of orthomodular lattices (posets) has been introduced as a "quantum logic", a significant question has been arisen: "Is the quantum logic really a logic?". This discussion especially commenced since the article [25] by Jauch and Piron in 1970. Based on their ideas, we can say a class of lattices is a logic when they satisfy the modus ponens law. By this condition, they deduced that it is very questionable to name the lattice of quantum mechanics as a logic. In 1971 and 1973 Greechie and Gudder also confirmed this idea in two articles [19] and [20]. Nevertheless in 1975, Hardegree argued in opposite to them saying there is at least one definable binary connective which has the essential conditions of material implication for orthomodular lattices [21]. This connective was used in some previous articles, for example [24], where it is called the Sasaki hook. Again in 1981, Hardegree discussed the minimal conditions for material implication in a different way and then he proved that the Sasaki hook is the best material implication on orthomodular lattices [23]. Using this connective, a complete axiomatic system was presented for orthomodular quantum logic, see [22]. In 2012, Foulis and Pulmannová introduced the Sasaki arrow for lattice effect algebra [18] which also was used in [29].

In the paper, we present some preliminaries and then in Section 2 we prove that this arrow is the best for lattice effect algebras. In Section 3 we build a fuzzy implication using the Sasaki arrow on the lattice effect algebras.

Moreover, several similar attempts concerning implications in orthomodular lattices and in lattice effect algebras were already published, for example, in [5, 6, 7, 8, 9, 10, 11, 12].

<sup>1</sup>A. Dvurečenskij is thankful for the support by the grants VEGA No. 2/0069/16 SAV and GAČR 15-15286S.

Download English Version:

<https://daneshyari.com/en/article/6856736>

Download Persian Version:

<https://daneshyari.com/article/6856736>

[Daneshyari.com](https://daneshyari.com)