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An application of fuzzy hypergraphs and hypergraphs in granular computing☆

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ABSTRACT

Granular computing emphasizes the exploitation of useful structures known as granular structures characterized by multi-level and multi-view. This paper studies the construction of granular structures models using fuzzy hypergraph and hypergraph, so that granules could be represented more intuitively and visually. In fuzzy hypergraph model and hypergraph model of granular computing, a vertex refers to an object, a fuzzy hyperedge or hyperedge corresponds to a granule. In particular, in the hypergraph model, the hyperedge is a partition of the universe, which is got from fuzzy equivalence relation. A fuzzy hypergraph or hypergraph relates to a set of granules and their relations in a specific granularity, and a series of hypergraphs correspond to a hierarchical structure. Based on granular structures, the mapping between fuzzy hypergraphs or hypergraphs presents the relations of the granules in different levels. The results show that it is efficient to represent the partition by means of fuzzy hypergraph and hypergraph, and it is a useful way to represent granular structures through fuzzy hypergraph model or hypergraph model.

1. Introduction

Granular computing (briefly GrC) is a label of theories, methodologies, techniques, and tools that take advantage of granules [2]. The basic ideas of granular computing have been explored in many fields, such as artificial intelligence, interval analysis, quantization, rough set theory, cluster analysis, machine learning, databases, and many others [33]. There are numerous fundamental issues in granular computing, such as granulation of the universe, description of granules, relationships between granules, and computing with granules. Various models of granular computing have been proposed and studied. For example, in 2002, a model of granular computing based on information tables was proposed [31], and within this model, various methods for the construction, interpretation, and representation of granules were examined. In 2012, Pedrycz, et al. introduced a concept of the granular representation of numeric membership functions of fuzzy sets [19], which offers a synthetic and qualitative view at fuzzy sets and their ensuing processing. Bisi et al. [1] investigated the mathematical foundations of indistinguishability relation through the introduction of two new structures which are a complete lattice and an abstract simplicity complex, and they show that these structures can be studied at both a micro granular and a macro granular level, and that are naturally related to the core and the reducts. It is important to note that when using the method

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of granular computing in the process of problem solving, one may consider different descriptions of the same problem as multi-level of granularity, and link multi-level descriptions together to form a hierarchical structure. While being focused on the different level descriptions of granularity, one can obtain various levels of knowledge, as well as inherent knowledge structure. Therefore, the granular computing approach deals with the construction of granules, computing and reasoning with granules, and the switching among different granularity, and there is a renewed and fast growing interest in granular computing [20,21,27,29,30].

Graph theory as a classical mathematical tool has numerous applications to problems in computer science, electrical engineering, system analysis, operations research, economics, networking routing, transportation, and so on. Recently, it is noteworthy that graph theory is also used in granular computing and the representation of granular structures in the filed of theoretic computer research. For example, Chiaselotti, et al. interpreted the adjacency matrix of a (finite) simple undirected graph G as a Boolean information table [4]. In this way, they used the theoretical framework developed in GrC-RS (for a detailed analysis concerning the links between GrC and rough sets) to find new properties concerning the graph G. Furthermore, Chiaselotti et al. [5] interpreted the adjacency matrix of any simple undirected graph G in terms of data information table, and based on the constructions, they provided a geometric characterization and determined their structure for basic graph families, which is one of the most studied structures in database theory. In 2012, Chen et al. studied the testing bipartiteness of simple undirected graphs based on a generalized rough set and defined a new binary relation induced from a simple undirected graph [7]. However, in many cases, the aspects of a graph theoretic problem may contain some uncertainty in some sense. For instance, vehicle travel time or vehicle capacity on a road network may not be known exactly, and in such cases, it is natural to deal with the uncertainty using the methods of fuzzy sets and fuzzy logic. In hypergraph, an hyperedge contains many vertices, and the vertices could be described a n-ary relation. Compared with graph, as an extension of graph, hypergraph provides more powerful description and analysis methods for solving real world problems in the modeling complex systems. Naturally, combining the advantages both fuzzy systems and graph models, a modeling system with fuzzy binary relation and multiary relations between objects, could be transformed into a fuzzy hypergraph. In fact, many researchers have considered to characterize granular computing from hypergraph perspective. Liu et al. [16] studied clustering on the large complex data sets by granular computing and hypergraph clustering algorithm, which extract the frequent item sets. Using granular computing method, these frequent item sets could be mapped into hyperedges in a hypergraph, then a multi-level hypergraph partitioning algorithm is used to divide the hypergraph into k parts. This process produced clusters from the larger complex data sets. Wong and Wu [28] used the idea of granularity and hypergraph method to investigate database scheme introduced by hypergraph. With the reducing of the circles of the hyperedges in hypergraph, the multi-valued dependency of the database is reduced. Finally, the hierarchical structure of granular database scheme is constructed by their algorithm. Chen et al. [2] proposed a hypergraph model of granular computing. In their model, a vertex refers to an object, a hyperedge corresponds to a granule, a hypergraph relates to a set of granules and their relations in a specific granularity, and a series of hypergraphs correspond to a hierarchical structure. Stell [23–26] investigated a suitable generalization of relations on a hypergraph or graph rather than relations on a set, which is the expansion of mathematical morphology from sets to graphs or hypergraphs. Based on their discussion, the hypergraph model is an effective representation method of granular structures and a convenient way for problem solving.

With the development of granular computing theory, different granular computing models have been revealed, among which there are three principal approximate computing models, fuzzy sets, rough sets and quotient space. Based on fuzzy sets, fuzzy equivalence relation was presented by Zadeh [34] as a generalization of equivalence relation on a finite domain. Then fuzzy equivalence class and partition were proposed and analyzed in detail by many scholars [29,39]. Karypis et al. [13] presented a hypergraph partitioning algorithm based on the multi-level paradigm, in which a sequence of successively coarser hypergraphs is constructed. It shows that a simple granular of the universe can be determined by a fuzzy equivalence relation or a partition, and using hypergraph model to represent the partition of the universe and granular structures is very useful.

A conceptual framework of granular computing was offered by weaving together three powerful ideas: structured thinking, structured problem solving, and structured information processing [30]. It emphasizes the exploitation of useful structures known as granular structures characterized by multi-level and multi-view. The multi-level requires that a granular structure consists of a family of integrative levels with different granularity, and the multi-view requires that it should be necessary to consider a family of multi-level structures with each representing a different view. A single hierarchical granular structure provides a multi-level understanding and representation of a problem or a system. But it typically captures one particular aspect and therefore offers one view. By constructing a family of hierarchies, it is possible to obtain multiple different views. Granular structures are a family of complementary hierarchies working together for a complete and comprehensive multi-view understanding and representation. Granular computing is built on the problem of this simplified granular structure. As we all know that many real world problems depicted and analyzed by graph theory have achieved good impacts. Based on the hypergraph, the model of granular structure is not merely a fair representation of the granular structure, but also can facilitate the establishment and conversion of the granular structure.

The ultimate objective of this study is to create a representation of granular structures by fuzzy hypergraphs and hypergraphs in such a way that granules could be represented more intuitively and visually, although previous researches have explored how to construct granules using hypergraph method in different fields [2,6,25]. In the fuzzy hypergraph model and hypergraph model, a vertex refers to an object, a fuzzy hyperedge or hyperedge corresponds to a granule. This representation is quite intuitively and visually. Since vertices and hyperedges of the hypergraph are defined by the way of set, the