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A new multiattribute decision making method based on multiplication operations of interval-valued intuitionistic fuzzy values and linear programming methodology

Shyi-Ming Chen*, Wen-Hsin Han

Department of Computer Science and Information Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan

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ABSTRACT

In this paper, we propose a new method for multiattribute decision making (MADM) using multiplication operations of interval-valued intuitionistic fuzzy values (IVIFVs) and the linear programming (LP) methodology. It can overcome the shortcomings of Chen and Huang's MADM method (2017), where Chen and Huang's MADM method has two shortcomings, i.e., (1) it gets an infinite number of solutions of the optimal weights of attributes when the summation values of some columns in the transformed decision matrix (TDM) are the same, resulting in the case that it obtains different preference orders (POs) of the alternatives, and (2) the PO of alternatives cannot be distinguished in some situations.

1. Introduction

The theory of interval-valued intuitionistic fuzzy sets (IVIFSs) [2] was proposed by Atanassov and Gargov, which is extended from the theory of intuitionistic fuzzy sets (IFSs) [1]. Some multiattribute decision making (MADM) methods [5–10,16,23,30–33,35–37] have been proposed based on IVIFSs. In [6], Chen and Huang proposed a method for MADM using interval-valued intuitionistic fuzzy values (IVIFVs) and the linear programming (LP) methodology [27]. However, in this paper, we find that the MADM method presented in [6] has two shortcomings, i.e., (1) it gets an infinite number of solutions of the optimal weights of attributes when the summation values of some columns in the transformed decision matrix (TDM) are the same, resulting in the case that it obtains different preference orders (POs) of alternatives, and (2) the PO of alternatives cannot be distinguished in some cases. In order to overcome the shortcomings of the method presented in [6], it is necessary to propose a new MADM method for overcoming the shortcomings of the MADM method presented in [6].

In this paper, we propose a new MADM method using multiplication operations of IVIFVs [35] and the LP methodology [27]. It can overcome the shortcomings of the MADM method presented in [6] for MADM in interval-valued intuitionistic fuzzy (IVIF) environments, where the proposed MADM method uses multiplication operations of IVIFVs to avoid to get an infinite number of solutions of the optimal weights of attributes when the summation values of some columns in the transformed decision matrix (TDM) are the same and uses Xu's score function [35] and Xu's accuracy function [35] to distinguish the PO between alternatives.

The remainder of this paper is arranged as follows. In Section 2, we present the preliminaries of this paper. In Section 3, we review the MADM method presented in [6]. In Section 4, we use some counter examples to illustrate the shortcomings

* Corresponding author.
E-mail address: smchen@mail.ntust.edu.tw (S.-M. Chen).

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of the MADM method presented in [6]. In Section 5, we propose a new MADM method using multiplication operations of IVIFVs and the LP methodology. In Section 6, we present the conclusions.

2. Preliminaries

Definition 2.1. [2]: An IVIFS A in the universe of discourse X can be represented by $A = \{< x_j, \mu_A(x_j), \upsilon_A(x_j) > | x_j \in X\}$, where $X = \{x_1, x_2, \ldots, x_n\}$, $\mu_A(x_j)$ and $\upsilon_A(x_j)$ are the membership degree and the non-membership degree of element x_j belonging to the IVIFS A, respectively, $\mu_A(x_j) = [p_j,\ q_j]$, $\upsilon_A(x_j) = [r_j,\ s_j]$, $0 \le p_j \le\ q_j \le 1$, $0 \le r_j \le\ s_j \le 1$, $q_j + s_j\ \le 1$ and $1 \le j \le n$.

Definition 2.2. [35]: The IVIFV of element x_j belonging to the IVIFS A is denoted by $([p_j,\ q_j],\ [r_j,\ s_j])$, where $0 \le p_j \le\ q_j \le 1$, $0 \le r_j \le\ s_j \le 1$, $q_j + s_j\ \le 1$ and $1 \le j \le n$.

Definition 2.3. [35]: Let $\tilde{\alpha} = ([p,\ q],\ [r, s])$ be an IVIFV, where $0 \le p \le q \le 1$, $0 \le r \le s \le 1$ and $q + s \le 1$. The multiplication operation between a positive constant λ and an IVIFV $\tilde{\alpha}$ is defined as follows:

$$\lambda\tilde{\alpha} = \left(\left[1 - (1-p)^{\lambda}, 1 - (1-q)^{\lambda}\right], \left[r^{\lambda}, s^{\lambda}\right]\right), \tag{1}$$

where $\lambda > 0$.

Definition 2.4. [35]: Let $\tilde{\alpha} = ([p,\ q],\ [r, s])$ be an IVIFV, where $0 \le p \le q \le 1$, $0 \le r \le s \le 1$ and $q + s \le 1$, and let S and H be the score function and the accuracy function of IVIFVs, respectively. The score value $S(\tilde{\alpha})$ and the accuracy value $H(\tilde{\alpha})$ of the IVIFV $\tilde{\alpha} = ([p,\ q],\ [r, s])$ are defined as follows:

$$S(\tilde{\alpha}) = \frac{p\ -\ r\ +\ q\ -\ s}{2}, \tag{2}$$

$$H(\tilde{\alpha}) = \frac{p +\ r\ +\ q +\ s}{2}, \tag{3}$$

where $S(\tilde{\alpha}) \in [-1,\ 1]$ and $H(\tilde{\alpha}) \in [0,\ 1]$. The larger the score value $S(\tilde{\alpha})$ of the IVIFV $\tilde{\alpha}$, the larger the IVIFV $\tilde{\alpha}$. If the score values of two IVIFVs are the same, then the larger the accuracy value of the IVIFV, the larger the IVIFV.

Definition 2.5. [35]: The interval-valued intuitionistic fuzzy weighted averaging (IVIFWA) operator g_w of the IVIFVs $\tilde{\alpha}_1$, $\tilde{\alpha}_2$, $\cdots$, and $\tilde{\alpha}_n$ is defined as follows:

$$\begin{aligned} g_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) &= \left(\left[1 - \prod_{j=1}^{n}\left(1-p_j\right)^{o_j}, 1 - \prod_{j=1}^{n}(1-q_j)^{o_j}\right], \left[\prod_{j=1}^{n} {r_j}^{o_j},\ \prod_{j=1}^{n} {s_j}^{o_j}\right]\right) \\ &= \left(\left[\rho^-, \rho^+\right],\ \left[\tau^-, \tau^+\right]\right), \end{aligned} \tag{4}$$

where $\tilde{\alpha}_j = ([p_j,\ q_j],\ [r_j,\ s_j])$, $0 \le p_j \le q_j \le 1$, $0 \le r_j \le s_j \le 1$, $0 \le q_j + s_j \le 1$, o_j represents the weight of the IVIFV $\tilde{\alpha}_j$, $o_j \in [0,\ 1]$, $1 \le j \le n$, $\sum_{j=1}^{n} o_j = 1$ $\rho^- = 1 - \prod_{j=1}^{n}(1-p_j)^{o_j}$, $\rho^+ = 1 - \prod_{j=1}^{n}(1-q_j)^{o_j}$, $\tau^- = \prod_{j=1}^{n} {r_j}^{o_j}$, $\tau^+ = \prod_{j=1}^{n} {s_j}^{o_j}$, $0 \le \rho^- \le \rho^+ \le 1$, $0 \le \tau^- \le \tau^+ \le 1$ and $0 \le \rho^+ + \tau^+ \le 1$.

3. A review of Chen and Huang's MADM method

Let $x_1, x_2, \cdots$, and x_m be alternatives, let $A_1, A_2, \cdots$, and A_n be attributes and let $\tilde{D} = (\tilde{d}_{ij})_{m\times n}$ be the decision matrix (DM) provided by the decision maker, where $\tilde{d}_{ij} = ([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])$ denotes an evaluating IVIFV of attribute A_j with respect to alternative x_i, $0 \le a_{ij}^- \le a_{ij}^+ \le 1$, $0 \le b_{ij}^- \le b_{ij}^+ \le 1$, $0 \le a_{ij}^+ + b_{ij}^+ \le 1$, $1 \le i \le m$ and $1 \le j \le n$. Let the weight of attribute A_j provided by the decision maker be represented by an IVIF weight $\tilde{o}_j$, where $\tilde{o}_j = ([h_j,\ y_j],\ [z_j,\ g_j])$ is an IVIFV, $0 \le h_j \le y_j \le 1$, $0 \le z_j \le g_j \le 1$, $0 \le y_j + g_j \le 1$ and $1 \le j \le n$. The MADM method presented in [6] is reviewed as follows:

Step 1: Based on Eq. (2) and the DM $\tilde{D} = (\tilde{d}_{ij})_{m\times n} = (([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+]))_{m\times n}$, compute the score value $m_{ij} = S(([a_{ij}^-,\ a_{ij}^+],\ [b_{ij}^-,\ b_{ij}^+])) = \frac{a_{ij}^-\ -\ b_{ij}^-\ +\ a_{ij}^+\ -\ b_{ij}^+}{2}$ of $\tilde{d}_{ij}$, where $m_{ij} \in [-1,\ 1]$, for constructing the transformed decision matrix (TDM) $M = (m_{ij})_{m\times n}$. Based on the obtained TDM $M = (m_{ij})_{m\times n}$, build the following LP model:

$$\max T = \sum_{i=1}^{m}\sum_{j=1}^{n}\left({o_j}^* \times m_{ij}\right),$$

where ${o_j}^*$ denotes the optimal weight of attribute A_j, $h_j \le {o_j}^* \le 1 - z_j$, $\sum_{j=1}^{n} {o_j}^* = 1$, ${o_j}^* \in [0,\ 1]$ and $1 \le j \le n$.

Step 2: Solve the LP model "$\max T = \sum_{i=1}^{m}\sum_{j=1}^{n}({o_j}^* \times m_{ij})$" to obtain the optimal weights ${o_1}^*$, ${o_2}^*$, $\cdots$, and ${o_n}^*$ of the attributes $A_1, A_2, \cdots$, and A_n, respectively, where $h_j \le {o_j}^* \le 1 - z_j$, ${o_j}^* \in [0,\ 1]$, $\sum_{j=1}^{n} {o_j}^* = 1$ and $1 \le j \le n$.