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Fuzzy alternating automata over distributive lattices

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ABSTRACT

Nondeterminism gives computation models the power of existential choice. As a generalization of nondeterminism, "alternation" gives computation models the power of existential choice and universal choice simultaneously. In this paper, we extend fuzzy nondeterministic automata to a model called fuzzy alternating automata over distributive lattices. Compared with the previous work, a weight labels a leaf node of the run tree rather than be involved in the edge between states when executing a transition. One advantage of our setting is that it is easy to complement a given fuzzy alternating automaton. It suffices to take the dual operation on the transition function and negate final costs on states. Moreover, we show that fuzzy alternating automata have the same expressive power as fuzzy nondeterministic automata, and the former ones are exponentially more succinct than the latter ones. In addition, we illustrate that such exponential blow-up is unavoidable.

1. Introduction

Nondeterminism has played important roles in the theory of computation [11]. In formal language theory [10,11], certain class of languages can be described by nondeterministic automata; in complexity theory [1,7,11,13], nondeterminism is the basis of NP-class of decision problems.

Alternation is a generalized notion of nondeterminism. Alternation gives the computing device the power of existential choice and universal choice during the course of a computation, whereas nondeterminism only gives the power of existential choice. In order to obtain a theoretical model of parallel computations, the notion of alternating automata was first put forward by Chandra et al. [4]. In [4], Chandra et al. also studied alternating Turing machines and alternating pushdown automata. Since then, alternating automata has already proven to be a useful model in studying formal verification [9,14,16,24,25].

In the study of linear temporal logic [24], Vardi simplified problems such as satisfiability of specifications and correctness of programs to ones about nonemptiness and containment of languages accepted by alternating automata. Vardi pointed out that alternating automata have the same expressive power as nondeterministic automata, and the former ones are exponentially more succinct than the latter ones. Compared with nondeterministic automata, alternating automata are easily to be complemented. Taking the dual operation on the transition function and exchanging final and non-final states, we can get a new alternating automaton accepting the complemented language with respect to the original alternating automaton.

To enhance the processing ability of alternating automata, Chatterjee et al. [5] firstly proposed a notion of weighted alternating automata over the set of real numbers. The expressive power and closure properties of weighted alternating automata in some special semantics such as Sup, LimSup, LimInf, LimAvg were shown in [5]. Comparing closure properties of weighted

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alternating automata and nondeterministic ones with respect to operators on languages such as Max, Min, Complement and Sum, Chatterjee et al. illustrated that alternating automata have more expressive power than nondeterministic ones in LimAvg and Discounted Sum-semantics. Shortly afterwards, Almagor and Kupferman [2] used positive Boolean formulas over the Cartesian product of the set of real numbers and the set of states to define another version of weighted alternating automata. In the Max-semantic, Almagor et al. also showed that alternating automata have more expressive power than nondeterministic ones. Almagor and co-workers [2,5] had already proposed weight alternating automata models, however for simplicity only automata with no final costs on states and the expressive power in some special semantics had been considered. The discussion of more general cases was not involved in [2,5].

In this paper, we put forward a notion of fuzzy alternating automata over distributive lattices using positive Boolean formulas over the union of a lattice and the set of states. We take the influence caused by final costs on states into consideration. Closure properties under union, intersection and complement do hold. In particular, for complement, the dual operation can be taken on transition functions effectively. Moreover, we show that fuzzy alternating automata and fuzzy nondeterministic ones have the same expressive power, while the former ones are exponentially more succinct than the latter ones.

The content of our paper is arranged as follows. In Section 2, we briefly recall some basic facts on alternating automata. In Section 3, we introduce a notion of fuzzy alternating automata. Afterwards, the equivalence relation between fuzzy alternating automata and fuzzy nondeterministic automata and closure properties of fuzzy alternating automata are given in Sections 3 and 4. In addition, we take an example to show the exponential blow-up arising from the translation from a fuzzy alternating automaton to a fuzzy nondeterministic automaton is unavoidable. At last, some problems could be considered later are shown in Section 5.

2. Preliminaries

In this section, we review a few notions and basic facts [3,4,8,10–12,15,24]. We write J for the index set and write $|Q|$ for the cardinal number of the set Q throughout the paper.

Let X be a set, we use $\mathcal{B}^+(X)$ to denote the set of positive Boolean formulas over X, i.e., Boolean formulas built by elements of X using $\wedge$ and $\vee$. In addition, we put the formulas **true** and **false** in $\mathcal{B}^+(X)$. For $Y \subseteq X$ and $\theta \in \mathcal{B}^+(X)$, if the value is true after assigning *true* to elements of Y and assigning *false* to elements of $X \backslash Y$, then we say that Y satisfies θ; furthermore, if no strict subset of Y satisfies θ, we say that Y satisfies θ in a minimal manner. Obviously, $\{x_2, x_3, x_4\}$ satisfies the formula $(x_1 \vee x_2) \wedge (x_3 \vee x_4 \vee x_5)$, and $\{x_1, x_3\}$, $\{x_2, x_5\}$ both satisfy this formula in a minimal manner, while $\{x_1, x_2\}$ and $\{x_3, x_5\}$ do not.

Before recalling some basic notions and results of alternating automata, we start by reviewing the notion of nondeterministic automata [10,11]. A nondeterministic automaton is a five-tuple $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$, where Q, Q_0 and F are nonempty finite sets of states, initial states and final states respectively, and Q_0, $F \subseteq Q$; Σ is a nonempty finite alphabet; δ is a transition function from $Q \times \Sigma$ into 2^Q, where 2^Q denotes the set of subsets of Q. A run r of $\mathcal{A}$ on a word $a_1 a_2 \ldots a_n \in \Sigma^*$ is a sequence of states $q_0, q_1, \ldots, q_n$ such that $q_0 \in Q_0$ and $q_{i+1} \in \delta(q_i, a_{i+1})$ for any $i = 0, \ldots, n-1$, where Σ^* denotes the set of finite strings over Σ including empty string ε. Moreover, if $q_n \in F$, then r is accepting. The language accepted by $\mathcal{A}$ is $L(\mathcal{A}) = \{w \in \Sigma^* | \delta^*(q_0, w) \cap F \neq \emptyset, q_0 \in Q_0\}$, where δ^* is an extension of δ satisfying: (1) $\delta^*(q, \varepsilon) = \{q\}$; (2) $\delta^*(q, w_1 a) = \bigcup_{q' \in \delta^*(q, w_1)} \delta(q', a)$ for $w_1 \in \Sigma^*$.

Let $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$ be a nondeterministic automaton. δ maps a state to a set of possible next states after inputting a symbol from Σ, which can be represented by some formulas from $\mathcal{B}^+(Q)$. For example, $\delta(q, a) = \{q_1, q_2, q_3\}$ can be written as $\delta(q, a) = q_1 \vee q_2 \vee q_3$. Based on this representation, the notion of alternating automata can be given formally.

An alternating automaton is a five-tuple $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$, where Q, Σ and F are the same as those in the nondeterministic automaton; $q_0 \in Q$ is the initial state; δ is a transition function from $Q \times \Sigma$ into $\mathcal{B}^+(Q)$. The language accepted by $\mathcal{A}$ is characterized by induction. For instance, suppose that $\delta(q, a) = (q_1 \wedge q_2) \vee q_3$ is a transition of $\mathcal{A}$, then $\mathcal{A}$ accepts the word aw from q, if it accepts the word w from both q_1 and q_2 or from q_3, where w is a word of Σ^*. Clearly, this transition includes both features of existential choice (the disjunction in the formula) and universal choice (the conjunction in the formula). In alternating automata, $\delta(q, a)$ can be an arbitrary formula from $\mathcal{B}^+(Q)$.

Indeed, an alternating automaton could have more than one initial states. Let $\mathcal{A} = (Q, \Sigma, \delta, Q_0, F)$ be a such automaton with $|Q_0| > 1$. We can convert $\mathcal{A}$ to another equivalent alternating automaton $\mathcal{B}$ with a unique initial state analogously to Li and Pedrycz [21,22]. The targeted automaton is $\mathcal{B} = (Q \cup \{q'\}, \Sigma, \delta', q', F)$ $(q' \notin Q)$, where $\delta'(q', a) = \bigvee_{q \in Q_0} \delta(q, a)$ and $\delta'(q, a) = \delta(q, a)$ for any $q \in Q$. Hence, it is sufficient to discuss alternating automata with a unique initial state in the following.

Because of the universal choice, a run of an alternating automaton is a tree. The notation $|x|$ denotes the level of node x, which is the distance from x to the root ε; in particular, $|\varepsilon| = 0$. A branch $\beta = x_0, x_1, \ldots$ of a tree is a maximal sequence of nodes, where x_0 is ε and x_i is the parent of x_{i+1} for any $i \geq 0$. In an alternating automaton, a run tree r of it is a Q-labeled tree and we use $r(x) = q$ to denote that q is the label of x of r.

Definition 2.1 (See [24]). Let $\mathcal{A} = (Q, \Sigma, \delta, q_0, F)$ be an alternating automaton. A run of $\mathcal{A}$ on a finite word $w = a_0 a_1 \ldots a_{n-1} (w \in \Sigma^*)$ is a finite tree r such that $r(\varepsilon) = q_0$ and the following holds: