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A stopping criterion for multi-objective optimization evolutionary algorithms

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ABSTRACT

This paper puts forward a comprehensive study of the design of global stopping criteria for multi-objective optimization. In this study we propose a global stopping criterion, which is terms as MGBM after the authors surnames. MGBM combines a novel progress indicator, called mutual domination rate (MDR) indicator, with a simplified Kalman filter, which is used for evidence-gathering purposes. The MDR indicator, which is also introduced, is a special-purpose progress indicator designed for the purpose of stopping a multi-objective optimization. As part of the paper we describe the criterion from a theoretical perspective and examine its performance on a number of test problems. We also compare this method with similar approaches to the issue. The results of these experiments suggest that MGBM is a valid and accurate approach.

1. Introduction

Most soft-computing, heuristic, non-deterministic or numerical methods all have in common that they need a stopping criterion. The stopping criterion, which is usually a heuristic itself, is responsible for minimizing the wastage of computational resources by detecting scenarios where it makes no sense to continue executing the method.

Stopping criteria can be grouped into local (iteration-wise) criteria and global (execution-wise) criteria. Local criteria have access only to data pertaining to each iteration of the method. They measure the difference between the current solution and a predefined reference or optimal value and then decide when they are close enough. This type of criterion has the obvious and paradoxical shortcoming of requiring a priori knowledge of the desired optimal value of the solution. This potential weakness has no significant impact if the class of problem being addressed allows the reference value to be replaced by the axis “zero” reference. This applies, for example, to function approximation and other types of problems that can be reduced to an error minimization problem.

Stopping criteria must keep track of the process progress across different iterations in order to make decisions relying on the long-term behavior of the algorithm being monitored. This evidence-gathering process has two positive impacts: (i) algorithm progress can be assessed in a relative fashion by comparing the outcome of different iterations and (ii) algorithm progress is more resilient to local optima and noise as it takes into account different iterations.

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Evolutionary algorithms (EAs) [4] are a class of population-based metaheuristic optimization methods. They also require a stopping criterion, but the vast majority of applications have bypassed this matter by using a termination scheme that specifies a finite number of iterations. Furthermore, although some works like [20,37,45,46] have addressed this issue does not appear to have propagated to the rest of the research community.

This is especially applicable to multi-objective optimization evolutionary algorithms (MOEAs) [9]. MOEAs are a type of evolutionary algorithm specially conceived for solving multi-objective optimization problems (MOPs) [17]. MOPs are optimization problems where two or more functions should be jointly optimized. The solution to these problems is a set, known as the Pareto-optimal set, which contains one or more feasible solutions, including the best trade-off values (either maximum or minimum) of the functions.

In the multi-objective case, stopping criteria approaches must apply relative improvement metrics that analyze the partial results of the algorithm across iterations. Therefore, there is no need to resort to an absolute comparison with an a priori established threshold. In the particular case of MOEAs, this type of criterion should compare the non-dominated fronts yielded by different iterations in order to determine how the optimization process is progressing.

The formulation of an effective criterion is particularly complex in the MOP case, as judging the optimization progress can turn out to be as complex as the optimization itself. In other types of problems, such as function approximation, pattern recognition or single-objective optimization, on the other hand, the axis can be used as a “zero” reference for progress measurement, as previously explained. This approach is inviable for MOPs since its solution is a set of points. Therefore, progress must be assessed in a relative manner using progress indicators rather than the actual solution set. There are a number of quality indicators that can be repurposed for this task, but their high computational cost is an obstacle to their application.

There has been little theoretical research dealing with MOEA convergence [19,36]. Probably on the above grounds, the formulation of an efficient stopping criterion for MOEAs and other MOP optimizers has been left aside, although it has been repeatedly named as one of the key topics in need of proper attention in the research area [7,8].

In this work we put forward a comprehensive study of the design of a global stopping criteria for multi-objective optimization. In particular we put forward an in-depth study of a global stopping criterion, which is termed as MGBM that was previously introduced by the authors [28,29]. MGBM combines a novel progress indicator, named mutual domination rate (MDR) indicator, with a simplified Kalman filter [22], which is used as an evidence-gathering process. The MDR indicator, which is also introduced here, is a special-purpose solution designed to deal with stopping. It is capable of gauging the progress of the optimization at a low computational cost and is therefore suitable for solving complex or many-objective problems.

The viability of the proposal is established by comparing it with some other possible alternatives. In particular, it is compared with the binary versions of the hypervolume indicator and the additive epsilon indicator [26] as progress indicators, and the application of statistical hypothesis testing to evidence assessment.

The theoretical and computational properties of the each of the components are discussed and contrasted. We also run a set of experimental tests. These tests are intended to assess each component combination under different circumstances in order to confirm that the method is capable of detecting “success” and “failure” stopping conditions. In these experiments we address some community-accepted test problems with the fast non-dominated sorting genetic algorithm (NSGA-II) [14], the improved strength Pareto evolutionary algorithm (SPEA2) [47], the Pareto envelope-based selection algorithm (PESA) [10], the multiobjective evolutionary algorithm based on decomposition (MOEA/D) [44], the *S*-metric selection evolutionary multiobjective optmization algorithm (SMS-EMOA) [5], the multi-objective optimization neural estimation of distribution algorithm (MONEDA) [31] and the multi-objective adaptive resonance theory estimation of distribution algorithm (MARTEDA) [30].

The main contributions of this paper can be summarized as:

- detailed discussion of the stopping criterion issue and its current state, requirements and problem-solving strategies;
- discussion of different approaches for addressing this issue, and;
- the proposal and testing of a novel stopping criterion.

The rest of this paper is organized as follows. In Section 2, we discuss the background. In Section 3, we dissect and analyze the stopping criteria issue in detail. In Section 4, we present the different components of our method, dealing with the local improvement determination and the evidence-gathering strategies. In Section 5, we discuss and present the experimental results that allow to review the properties of the criteria from a practical perspective. Finally, in Section 6 we outline some concluding comments and remarks, and outline future lines of work.

2. Multi-objective optimization

The concept of multi-objective optimization refers to the process of finding one or more feasible solutions to a problem by trading off the equally optimal values of two or more functions subject to a set of constraints.

Stated more formally, a multi-objective optimization problem (MOP) can be defined as: