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A grid-based adaptive multi-objective differential evolution algorithm☆

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ABSTRACT

Differential evolution is an excellent optimizer for single objective optimization problems. To extend its use for multi-objective optimization problems with promising performance, this paper proposes a grid-based adaptive multi-objective differential evolution algorithm. The main feature of the proposed algorithm is its dynamical adjustment of convergence and diversity by exploiting the feedback information during the evolutionary process. In the algorithm, the objective space is divided into grids according to the nondominated solutions in the population. Based on the grid, three indexes including grid fitness, grid density, and grid-objective-wise standard deviation are defined to measure individual rank, individual density, and population search status quo, respectively. Afterwards, three main components of the algorithm, i.e., parents selection, parameter control, and population update, are implemented based on grid index values. To validate algorithm performance, comprehensive experiments are carried out on thirty-one benchmark problems. The results show that the proposed algorithm outperforms nine state-of-the-art competitors in terms of three performance metrics. Also, the effectiveness of three components and the sensitivity of two design parameters in the algorithm are empirically quantified.

1. Introduction

In real world, we often encounter problems with more than one objective to be simultaneously optimized termed as Multi-objective Optimization Problems (MOPs). The solution of a MOP is a set of Pareto optimal solutions, in which each one is with different trade-off among objectives. Evolutionary algorithms, using mechanisms inspired by various biological metaphors such as reproduction, mutation, recombination, and selection, search for well approximate solutions to all types of problems because they ideally do not make any assumption about the underlying fitness landscape of problems. Nowadays, Multi-Objective Evolutionary Algorithms (MOEAs) have emerged as prevalent and effective approaches for tackling MOPs because they can handle problems with various characteristics and generate the solution set in a single run.

The performance of a MOEA is primarily determined by two main factors, algorithm structure and search operators. For the former one, three main types of algorithm structures have been investigated in literature, i.e., Pareto-based algorithms

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[7], indicator-based algorithms [38], and decomposition-based algorithms [23]. The latter one determines the search power of an algorithm. Currently, most MOEAs use genetic algorithm (GA) as their search operators.

Differential Evolution (DE) is a different meta-heuristic with features of few control parameters and simple implementation [37], and it has shown great success in single-objective optimization problems. More importantly, DE is able to generate solutions with significant dissimilarity in their decision vectors, which is quite useful for simultaneously searching multiple nondominated solutions in MOPs. However, when applying DE to MOPs, two challenges arise. The first is the preservation of diverse nondominated solutions, which is the common issue in any MOEAs. The other one, arisen from DE alone, is DE rarely converges but stagnants early due to a relatively high diversity during evolutionary process, which results in a diverse yet not well-converged solution set.

Maintaining a delicate balance between convergence and diversity is the primary challenge when designing a MOEA. Among existing MODEs, most algorithms focus on a particular facet, while few algorithms are devoted to balancing convergence and diversity through the design of synergistic components using feedback information through evolutionary process. To exploit feedback information, a proper tool should be applied. The grid, which maps the objective space from a continuous domain into a discrete domain, provides a simple yet efficient tool to quantify individual performance and evolution progress. The grid has already shown its benefit in some MOEAs [5,33]; however, its use mainly restricted on maintaining a diverse solution set, rather than providing feedback information for dynamically adjusting algorithm behavior. In this paper, by exploiting the utilities of grid, a novel algorithm called Grid-based Adaptive MODE (in short for GAMODE) with promising performance is proposed. In the algorithm, the objective space is divided into grids according to the nondominated solutions in the population. Then three grid-based indexes, i.e., Grid Fitness (*GF*), Grid Density (*GD*), and Grid-Objective-Wise Standard Deviation (*GOWSD*), are defined to measure individual rank, individual density, and population search progress, respectively. Afterwards, three main components of GAMODE, including parents selection, parameter setting, and population update, are devised under the same grid environment. Since the grid setting varies with the nondominated solutions in the population, the algorithm can quickly adapt to the change of search progress. Therefore, a reasonable balance between convergence and diversity can be achieved.

The main contribution of this work can be summarized as follows:

- A new MOEA with promising performance using DE as search operator is proposed. Rather than devising a new MODE by considering one aspect, the proposed GAMODE is implemented by simultaneously integrating its main components under the same framework, i.e., the grid.
- The utilities of grid are thoroughly exploited in GAMODE via three grid indexes, i.e., *GF*, *GD*, and *GOWSD*. Based on *GOWSD*, the population is divided into two subpopulations, each focusing on a certain task. During parents selection, individual performance and its position in decision space are considered. A double-level parameter adaptation strategy is designed to adaptively adjust the scale factor and crossover rate of each individual. In addition, *GD* index is utilized as a second criterion to update population with the aim of maintaining a well-distributed solution set.
- Extensive experiments have been conducted on four well-known test suites including Zitzler-Deb-Thiele (ZDT), Deb-Thiele-Laumanns-Zitzler (DTLZ), Walking Fish Group (WFG) and UF test suite by using three performance metrics. The results show that GAMODE outperforms nine state-of-the-art MOEAs. Also, the effectiveness of three components and the sensitivity of two design parameters in GAMODE are empirically quantified.

The rest of this paper is organized as follows. In Section 2, the concept of DE and the paradigm of a general MODE are presented. In Section 3, a brief review on MODEs and the motivation of this work are outlined. Section 4 expounds the GAMODE algorithm in detail, followed by the experimental results in Section 5. Conclusions are drawn in Section 6.

2. Preliminary

DE is a population-based stochastic optimizer for single objective optimization problems in continuous domain, distinguished by its unique mutation mechanism. During the evolution process, DE maintains a population P at the tth iteration, consisting of NP individuals with dimension D, i.e., $P^t = \{\boldsymbol{x}_1^t, \boldsymbol{x}_2^t, \ldots, \boldsymbol{x}_{NP}^t\}$ with $\boldsymbol{x}_i^t = \{x_{i,1}^t, x_{i,2}^t, \ldots, x_{i,D}^t\}$. After initializing P^t(i.e., $t = 0$), the evolution begins with the application of mutation operator, that is, for each individual $\boldsymbol{x}_i^t$ (called target vector), a mutant individual $\boldsymbol{v}_i^t$ (called donor vector) is generated via mutation operation. The classical “rand/1” mutation is formulated as

$$\boldsymbol{v}_i^t = \boldsymbol{x}_{r1}^t + F \cdot (\boldsymbol{x}_{r2}^t - \boldsymbol{x}_{r3}^t), \tag{1}$$

where $r1$, $r2$, and $r3$ are distinct indexes sampled from $\{1, 2, \ldots, NP\}/\{i\}$; $F \in (0, 1)$ is a parameter called scale factor.

Following mutation, the donor vector $\boldsymbol{v}_i^t$ recombines with the target vector $\boldsymbol{x}_i^t$ to produce an offspring $\boldsymbol{u}_i^t$ (called trial vector) by crossover operation, depicted as

$$u_{i,j}^t = \begin{cases} v_{i,j}^t, & \text{if } \texttt{rand}_j(0,1) \le \mathrm{Cr} \text{ or} j = j_{rand} \\ x_{i,j}^t, & \text{otherwise} \end{cases}, \tag{2}$$

where $Cr \in (0, 1)$ is crossover rate and $j_{rand} \in \{1, 2, \ldots, D\}$ is a random integer.