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Improving the modified interval linear programming method by new techniques

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ABSTRACT

In this study, we consider interval linear programming (ILP) problems, which are used to deal with uncertainties resulting from the range of admissible values in problem coefficients. In most existing methods for solving ILP problems, a part of the solution region is not feasible. The solution set obtained through the modified ILP (MILP) method is completely feasible (i.e., it does not violate any constraints), but is not completely optimal (i.e., some points of the region are not optimal). In this paper, two new ILP methods and their sub-models are presented. These techniques improve the MILP method, giving a solution region that is not only completely feasible, but also completely optimal.

1. Introduction

Uncertainties in many real-world problems mean that their parameters may be specified as lying between lower and upper bounds. To deal with such uncertainties, interval linear programming (ILP) is used. Researchers working on ILP problems have proposed several methods for solving ILP models [1–34]. Some have used interval arithmetic and extensions of the simplex algorithm [3,16,17,23], whereas others have focused on basis stability [9,18,25,30]. Under the assumption of basis stability, it is possible to obtain the optimal solution set of ILP. The ILP model has sometimes been divided into two sub-models to obtain the solution set [10,13,15,29,32–34].

In [2], the authors obtained the optimal solution set to an ILP problem by using the best and worst problem constraints when all components of the optimal solutions to the ILP model are positive. This assumption can be derived by solving the best problem. If all components of the feasible solution to the best problem are positive, then the feasible solution components (and hence the optimal solution components) in all of the characteristic models (and thus the ILP model) are positive.

In the best and worst cases (BWC) method proposed by Tong [32], the ILP model was converted into two sub-models, the best and worst sub-models, which have the largest and smallest feasible regions, respectively. A given point is feasible for the ILP model if it satisfies the constraints of the best problem, and it is optimal for the ILP model if it is optimal for at least one characteristic model. The BWC method has been extended by Chinneck and Ramadan to ILP models with equality constraints [4]. A novel ILP method was proposed by Huang and Moore [15].

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The solutions given by the BWC and ILP methods may stray into infeasible regions. Although the BWC method introduces exact bounds for the values of the objective function, the solution could exist in an area that is infeasible. To guarantee that the given solution is completely feasible, Zhou et al. [34] proposed the modified ILP method (MILP), in which an extra constraint is added to the second sub-model. However, the solutions resulting from MILP may not be optimal.

In this paper, we propose improved ILP (IILP) and improved MILP (IMILP) methods for solving ILP problems. The solutions to these methods are guaranteed to be completely feasible and completely optimal. The feasibility and optimality are illustrated by some numerical examples.

In the following, an interval number $x^{\pm}$ is represented as $[x^-, x^+]$, where $x^- \le x^+$. If $x^- = x^+$, then $x^{\pm}$ is degenerate. $x^{\pm} \ge 0$ if and only if $x^+ \ge 0$ and $x^- \ge 0$. In addition, $x^{\pm} \le 0$ if and only if $x^+ \le 0$ and $x^- \le 0$. If A^- and A^+ are two matrices in $\mathbb{R}^{m\times n}$ such that $A^- \le A^+$, then the set of matrices

$$\mathbf{A}^{\pm} = [A^-, A^+] = \{A \mid A^- \le A \le A^+\}$$

is called an interval matrix, and the matrices A^- and A^+ are called its bounds. Center and radius matrices are defined as

$$A^c = \frac{1}{2}(A^+ + A^-), \quad \Delta_{\mathbf{A}^{\pm}} = \frac{1}{2}(A^+ - A^-).$$

A square interval matrix $\mathbf{A}^{\pm}$ is said to be regular if each $A \in \mathbf{A}^{\pm}$ is non-singular.

A special case of an interval matrix is an interval vector $\mathbf{x}^{\pm} = \{\mathbf{x} \mid \mathbf{x}^- \le \mathbf{x} \le \mathbf{x}^+\}$, where $\mathbf{x}^-, \mathbf{x}^+ \in \mathbb{R}^n$. Interval arithmetic has been studied in [1].

2. Overview of ILP model solving methods

In this section, we review some methods for solving ILP models with inequality constraints. Models with equality constraints have also been investigated [11,29]. Consider the following ILP:

$$\begin{aligned} \max \quad & z^{\pm} = \sum_{j=1}^{n} c_j^{\pm} x_j^{\pm} \\ s.t. \quad & \sum_{j=1}^{n} a_{ij}^{\pm} x_j^{\pm} \le b_i^{\pm}, \quad i = 1, 2, \ldots, m \\ & x_j^{\pm} \ge 0, \quad j = 1, 2, \ldots, n. \end{aligned} \tag{1}$$

The characteristic model of the ILP model (1) is

$$\begin{aligned} \max \quad & z = \sum_{j=1}^{n} c_j x_j \\ s.t. \quad & \sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1, 2, \ldots, m \\ & x_j \ge 0, \quad j = 1, 2, \ldots, n, \end{aligned} \tag{2}$$

where $a_{ij} \in a_{ij}^{\pm}$, $c_j \in c_j^{\pm}$, and $b_i \in b_i^{\pm}$.

The feasible solution set of the ILP is defined as $\{\mathbf{x} \in \mathbb{R}^n : \sum_{j=1}^{n} a_{ij}^- x_j \le b_i^+,\ x_j \ge 0, i = 1, \ldots, m, j = 1, \ldots, n\}$.

Moreover, the optimal solution set of the ILP is defined as the set of all optimal solutions over all characteristic models.

According to [32], the BWC method can be used to solve model (1). The two sub-models are as follows:

Sub-model 1 (the best problem).

$$\begin{aligned} \max \quad & z^{+} = \sum_{j=1}^{n} c_j^{+} x_j \\ s.t. \quad & \sum_{j=1}^{n} a_{ij}^{-} x_j \le b_i^{+}, \quad i = 1, 2, \ldots, m, \\ & x_j \ge 0, \quad j = 1, 2, \ldots, n. \end{aligned} \tag{3}$$

Sub-model 2 (the worst problem).

$$\begin{aligned} \max \quad & z^{-} = \sum_{j=1}^{n} c_j^{-} x_j \\ s.t. \quad & \sum_{j=1}^{n} a_{ij}^{+} x_j \le b_i^{-}, \quad i = 1, 2, \ldots, m, \\ & x_j \ge 0, \quad j = 1, 2, \ldots, n. \end{aligned} \tag{4}$$

Theorem 2.1 [32]. *If z_{opt}^+ and z_{opt}^- are the best and worst optimal values of the objective function, respectively, then all of the optimal values of the objective function lie in $[z_{opt}^-, z_{opt}^+]$.*

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