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Fault-tolerant edge-bipancyclicity of faulty hypercubes under the conditional-fault model

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ABSTRACT

It is well-known that the n-dimensional hypercube Q_n is one of the most versatile and efficient interconnection network architecture yet discovered for building massively parallel or distributed systems. Let F be the faulty set of Q_n and let f_v, f_e be the numbers of faulty vertices and faulty edges in F, respectively. An edge $e=(x,y)$ is said to be *free* if e, x, y are not in F, and a cycle is said to be *fault-free* if there is no faulty vertex or faulty edge on the cycle. In this paper, we prove that each free edge (x, y) in Q_n for $n \geq 3$ lies on a fault-free cycle of any even length from 6 to $2^n - 2f_v$ if $f_v + f_e \leq 2n-5$, $f_e \leq n-2$ and both x and y are incident to at least two free edges. This result confirms a conjecture reported in the literature.

1. Introduction

A multicomputer system is comprised of a plurality of processors that communicate by exchanging messages via an interconnection network (network for short). It is well-known that a topological structure of a network can be modeled by a loopless undirected graph G, where the vertex set $V(G)$ represents the processors and the edge set $E(G)$ represents the communication links. In this paper, we use graphs and networks interchangeably.

Let G be a graph. A *walk* in G is a sequence of vertices $(x_0, x_1, x_2, \ldots, x_k)$ such that $(x_i, x_{i+1}) \in E(G)$ for $0 \leq i \leq k-1$. If $x_i \neq x_j$ for every $i \neq j \in \{0, 1, \ldots, k\}$, then the walk is a *path* of length k from x_0 to x_k, and if further $x_0 = x_k$ exceptionally, then the walk is called a *cycle* of length k. We use $P[x, y]$ to denote a path from x to y, called an (x, y)-*path*. Two paths are vertex-disjoint if they have no common vertices. A path (or a cycle, resp.) of length k is called a k-*path* (or a k-*cycle*, resp.). A k-cycle is even or odd depending on the parity of k. By P and C, we commonly denote a path and a cycle respectively, and we denote the length of P and C by $|P|$ and $|C|$, respectively. The *distance* between x and y in G is denoted by $d_G(x, y)$, which is the length of a shortest path between x and y in G. Two graphs G and H are *isomorphic*, denoted by $G \cong H$, if there is a bijection ϕ between $V(G)$ and $V(H)$ such that for any $u, v \in V(G)$, $(u, v) \in E(G)$ if and only if $(u^\phi, v^\phi) \in E(H)$.

There are a lot of mutually conflicting requirements in designing the topology of a network. It is almost impossible to design a network which is optimum from all aspects. One has to design a suitable network depending on the requirements and its properties. One of the central issues in designing and evaluating an interconnection network is to study how well other existing networks can be embedded into this network. This problem can be modeled by the following *graph embedding problem*. Given a host graph H which represents the network into which other networks are to be embedded, and a guest graph G which

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represents the network to be embedded, the problem involves finding a mapping φ from $V(G)$ to $V(H)$ such that each edge of G can be mapped to a path in H. Two common measures of effectiveness of an embedding are the dilation, which measures the slowdown in the new architecture, and the load factor, which gauges the processor utilization. A graph embedding has two major applications: transplanting parallel algorithms developed for one network into a different one, and allocating concurrent processes to processors in the network [30,32].

If a guest graph G is isomorphic to a subgraph H' of the host graph H, then any isomorphism map from G to H' can be viewed as a special type of embedding, called *isomorphic embedding*. An isomorphic embedding is the most important embedding since such an embedding has both dilation and load one. As two common guest graphs, paths (i.e, linear arrays) and cycles (i.e., rings) are two fundamental networks for parallel and distributed computation which are suitable for developing simple algorithms with low communication cost. Many efficient algorithms were originally designed based on paths and/or cycles for solving a variety of algebraic problems, graph problems and some parallel applications, such as those in image and signal processing [1,3,7,12,30,32]. Thus, it is important to have an effective path and/or cycle embedding in a network.

The cycle embedding problem can also be briefly stated as follows: How to find a cycle of given length into a given graph. If a graph G contains cycles of every length from 3 to $|V(G)|$, then it is *pancyclic*, and it is *bipancyclic* if it contains a cycle of every even length from 4 to $|V(G)|$, where $|V(G)|$ is the number of vertices of G. The pancyclicity is an important measurement to determine whether a network is suitable for an application mapping rings of any length into the network [10,13,32]. In a heterogeneous computing system, each edge and each vertex may be assigned with distinct computing power and distinct bandwidth, respectively [28]. Thus, it is worthwhile to extend pancyclicity to edge-pancyclicity and vertex-pancyclicity [4,5,32,33]. If every edge (or vertex, resp.) of G lies on a cycle of every length form 3 to $|V(G)|$, then G is said to be *edge-pancyclic* (or *vertex-pancyclic*, resp.) and it is *edge-bipancyclic* (or *vertex-bipancyclic*, resp.) if every edge (or vertex, resp.) lies on a cycle of every even length form 4 to $|V(G)|$. A *bipartite graph* is one whose vertex set can be partitioned into two nonempty subsets X and Y such that every edge has one end-vertex in X and the other in Y. Bipancyclicity (or edge-bipancyclicity, vertex-bipancyclicity, resp.) is essentially a restriction of the concept of pancyclicity (or edge-pancyclicity, vertex-pancyclicity, resp.) to bipartite graphs whose cycles are necessarily of even length.

Element (edge or vertex) failure is inevitable when an interconnection network is put in use. Therefore, the fault-tolerant capacity of a network is a critical issue. The problem of fault-tolerant cycle embedding of some networks has received much attention recently [3,6,7,10,11,13,15,16,18–27,31,32,35]. Let $F = F_v \cup F_e$ be a faulty set of a graph G, where $F_v \subseteq V(G)$ and $F_e \subseteq E(G)$. Write $f_v = |F_v|$ and $f_e = |F_e|$. A vertex or an edge of G is *fault* if it is contained in F, and *fault-free* if it is not in F. Note that a faulty edge may have fault-free end-vertices and a fault-free edge may have faulty end-vertices. An edge (x, y) is said to be *free* if it is fault-free and both its end-vertices are fault-free (see [3, Fig. 1]). A path (or a cycle) is *fault-free* if it contains neither a faulty vertex nor a faulty edge. We use $G - F$ to denote the subgraph of G which is induced by $\{x \mid x \in V(G)$ and $x \notin F\}$ and does not contain the edges in F. Hence, an edge in $G - F$ is indeed a free edge of G. A graph G is *fault-tolerant bipancyclic* (or *fault-tolerant edge-bipancyclic*, *fault-tolerant vertex-bipancyclic*, resp.) if $G - F$ is bipancyclic (or edge-bipancyclic, vertex-bipancyclic, resp.).

An n-dimensional hypercube Q_n (an n-cube for short) is one of the most versatile and efficient architecture yet discovered for building massively parallel or distributed systems. It possesses quite a few excellent properties such as recursive structure, regularity, symmetry, small diameter, relatively short mean internode distance, low degree, and much small link complexity, which are very important for designing massively parallel or distributed systems [2,3,7,9,10,12,17,30]. One problem of fault-tolerant cycle embedding in faulty hypercubes is to determine whether a fault-free cycle of any possible length can be embedded in hypercubes, that is, the fault-tolerant bipancyclicity of hypercubes [3,24,25,30,32,34]. Here, Q_n can have only faulty vertices, only faulty edges or both faulty vertices and faulty edges. There is also an interesting fault-tolerant cycle embedding problem which is the fault-tolerant edge-bipancyclicity of hypercubes [13,18,22–26,30,31].

The fault-tolerant bipancyclicity of hypercubes has received a great deal of attention in recent years [3,24,32,34]. Under the conditional-fault model, that is, each fault-free vertex is incident to at least two free edges, Yang et al. [34] proved that there exists a fault-free cycle of any even length from 4 to 2^n in a faulty n-cube Q_n with $n \geq 3$ if $f_e \leq 2n - 5$ and $f_v = 0$. Later Tsai [24] proved that there exists a fault-free cycle of any even length from 4 to $2^n - 2f_v$ in a faulty n-cube Q_n with $n \geq 5$ if $f_v + f_e \leq 2n - 4$ and $f_e \leq n - 2$. Recently, Chang and Hsieh [3] generalized both results by proving that there exists a fault-free cycle of any even length from 4 to $2^n - 2f_v$ in a faulty n-cube Q_n with $n \geq 5$ if $f_v + f_e \leq 2n - 4$, $f_e \leq 2n - 5$ and each vertex is incident to at least two edges in which each of them is either fault-free or faulty with at least one faulty end-vertex.

For the fault-tolerant edge-bipancyclicity of hypercubes, it has also been investigated by many researchers in recent years [10,18,19,22–26,29,31,32]. Under the conditional-fault model, Xu et al. [31] proved that each edge in $Q_n - F$ with $n \geq 4$ lies on a fault-free cycle of any even length from 6 to 2^n if $f_v = 0$ and $f_e \leq n - 1$; Tsai [22] proved that each edge in $Q_n - F$ with $n \geq 3$ lies on a fault-free cycle of any even length from 4 to $2^n - 2f_v$ if $f_v \leq n - 2$ and $f_e = 0$, and [23] that each edge in $Q_n - F$ with $n \geq 4$ lies on a fault-free cycle of any even length from 6 to $2^n - 2f_v$ if $f_v = n - 1$ and $f_e = 0$. In this paper, under the conditional-fault model, we prove that each edge in $Q_n - F$ with $n \geq 3$ lies on a fault-free cycle of any even length from 6 to $2^n - 2f_v$ if $f_v + f_e \leq 2n - 5$ and $f_e \leq n - 2$. This result, together with Xu et al's result in [31], confirms a conjecture proposed in [22].

As a comparative viewpoint, for the fault-tolerant vertex-bipancyclicity of hypercubes, one may refer to [23,32].

2. Preliminaries

An *n-dimensional hypercube* Q_n is an undirected graph with 2^n vertices each labeled with an n-bit binary string $x_n x_{n-1} \ldots x_2 x_1$, where $x_i = 0$ or 1 for each $1 \leq i \leq n$, and with two binary strings adjacent if they have exactly