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Weighted fuzzy interpolative reasoning for sparse fuzzy rule-based systems based on piecewise fuzzy entropies of fuzzy sets

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ABSTRACT

In this paper, we propose a new method for weighted fuzzy interpolative reasoning in sparse fuzzy rule-based systems based on piecewise fuzzy entropies of fuzzy sets. First, the proposed method uses the representative values of antecedent fuzzy sets, the representative values of observation fuzzy sets, and the representative values of consequence fuzzy sets of fuzzy rules to get the characteristic points of the fuzzy interpolative result represented by a fuzzy set. Then, it calculates the piecewise fuzzy entropies between any two characteristic points of the antecedent fuzzy sets, the piecewise fuzzy entropies between any two characteristic points of the observation fuzzy sets, and the piecewise fuzzy entropies between any two characteristic points of the consequence fuzzy sets of the fuzzy rules, respectively. Then, it calculates the weights of the antecedent fuzzy sets of each fuzzy rule, respectively, and calculates the weight of each fuzzy rule. Then, it calculates the piecewise fuzzy entropies between any two characteristic points of the fuzzy interpolative result. Finally, it uses the secant method to calculate the degree of membership of each obtained characteristic point of the fuzzy interpolative result. The experimental results show that the proposed method outperforms the existing methods for dealing with the multivariate regression problems, the Mackey–Glass chaotic time series prediction problem, and the time series prediction problems.

1. Introduction

Fuzzy interpolative reasoning is a very important research topic for sparse fuzzy rule-based systems. In recent years, some fuzzy interpolative reasoning methods have been presented [1–3,7,8,10–27,34–39,42–48,50,52,55,57,58,60–68] for sparse fuzzy rule-based systems. Table 1 shows a comparison of the characteristics of the existing fuzzy interpolative reasoning methods.

In [1], Baranyi et al. presented a fuzzy interpolative reasoning method, called the General method, for sparse fuzzy rule-based systems. In [38], Huang and Shen presented a fuzzy interpolative reasoning method, called the HS method, for sparse fuzzy rule-based systems, where they constructed intermediate fuzzy rules and used the scale and move transformations to guarantee the fuzzy interpolative reasoning result to be a normal and convex fuzzy set. In [7], Chang et al. pointed out that the drawback of the HS method [38] is that the HS method is logically inconsistent with respect to the ratios of fuzziness, and they presented a fuzzy interpolative reasoning method, called the CCL method, based on the areas of fuzzy sets. The CCL method uses the areas of fuzzy sets and the ratios of fuzziness for fuzzy interpolative reasoning and uses the weighted average

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Table 1
A comparison of the characteristics of the existing fuzzy interpolative reasoning methods.

Methods	Characteristics
Fuzzy interpolative reasoning methods presented in [1,7,10,11,14,15,20,35,38,39,61] and [62]	Based on polygonal fuzzy sets or bell-shaped fuzzy sets, which can deal with fuzzy rules interpolation with multiple antecedent variables
Fuzzy interpolative reasoning methods presented in [27,63,64,65] and [67]	Deal with adaptive fuzzy interpolation
Fuzzy interpolative reasoning method presented in [68]	Deals with the closed form fuzzy interpolation
Fuzzy interpolative reasoning methods presented in [42] and [43]	Deal with the backward fuzzy interpolation
Fuzzy interpolative reasoning methods presented in [11,16,17,18,23] and [24]	Based on interval type-2 fuzzy sets
Fuzzy interpolative reasoning methods presented in [1,2,3,20,27,34,38,42,45,48,52,57,60,61] and [62]	Based on the two-fuzzy-rules fuzzy interpolation scheme
Fuzzy interpolative reasoning methods presented in [14,15,22,26,39,44,46,55,58,65,66] and [67]	Based on the multiple fuzzy rules interpolation scheme
Fuzzy interpolative reasoning methods presented in [8,14,15,24] and [50]	Deal with weighted fuzzy interpolative reasoning based on the weighted antecedent variables interpolation scheme
Fuzzy interpolative reasoning methods presented in [21] and [25]	Based on rough-fuzzy sets [5]
Fuzzy interpolative reasoning methods presented in [42] and [48]	Based on ranking values of fuzzy sets

method to deduce the fuzzy interpolative reasoning result. In [14], Chen and Chang presented a weighted fuzzy interpolative reasoning method, called the CC method, based on GA-based weights-learning techniques. In [20], Chen and Ko presented a fuzzy interpolative reasoning method, called the CK method, based on α-cuts of fuzzy sets and transformation techniques. In [52], Qiao et al. presented a fuzzy interpolative reasoning method, called the QMY method, for sparse fuzzy rule-based systems. However, the methods presented in [1,7,14,20,38] and [52] have the drawback that their forecasting accuracy rates are not good enough for dealing with the multivariate regression problems [29], the Mackey–Glass chaotic time series prediction problem [28] and the time series prediction problems [6,40]. Therefore, we need to develop a new fuzzy interpolative reasoning method for sparse fuzzy rule-based systems to overcome the drawback of the methods presented in [1,7,14,20,38] and [52] for increasing the forecasting accuracy rates.

In this paper, we propose a new method for weighted fuzzy interpolative reasoning based on piecewise fuzzy entropies of fuzzy sets. First, the proposed method uses the representative values [38] of the antecedent fuzzy sets A_{kj}, the representative values of the observation fuzzy sets A_j^*, and the representative values of the consequence fuzzy sets B_k of the fuzzy rule *Rule k* to get the characteristic points of the fuzzy interpolative result B^* represented by a fuzzy set, where $1 \leq k \leq p$, p is the number of fuzzy rules, $1 \leq j \leq m$, and m is the number of antecedent variables appearing in the antecedent of fuzzy rules. Then, it calculates the piecewise fuzzy entropies between any two characteristic points of the antecedent fuzzy sets A_{kj}, the piecewise fuzzy entropies between any two characteristic points of the observation fuzzy sets A_j^*, and the piecewise fuzzy entropies between any two characteristic points of the consequence fuzzy sets B_k of the fuzzy rule *Rule k*, respectively, where $1 \leq k \leq p$ and $1 \leq j \leq m$. Then, it calculates the weights of the antecedent fuzzy sets of each fuzzy rule, respectively, and calculates the weight of each fuzzy rule. Then, it calculates the piecewise fuzzy entropies between any two characteristic points of the fuzzy interpolative result B^*. Finally, it uses the secant method [9] to calculate the degree of membership of each obtained characteristic point of the fuzzy interpolative result B^*. We also apply the proposed method to deal with the multivariate regression problems [29], the Mackey–Glass chaotic time series prediction problem [28] and the time series prediction problems [6,40]. The experimental results show that the proposed method outperforms the existing methods [1,7,14,20,38,52].

The main contribution of this paper is that we propose a weighted fuzzy interpolative reasoning method for sparse fuzzy rule-based systems based on piecewise fuzzy entropies of fuzzy sets. The proposed weighted fuzzy interpolative reasoning method can overcome the drawbacks of the methods presented in [1,7,14,20,38] and [52]. It outperforms the existing methods [1,7,14,20,38,52] for dealing with the multivariate regression problems, the Mackey–Glass chaotic time series prediction problem and the time series prediction problems.

The rest of this paper is organized as follows. In Section 2, we briefly review the definitions of polygonal fuzzy sets [17] and bell-shaped fuzzy sets [17], briefly review the definition of the representative value [39] of a fuzzy set, present the definition of the closest fuzzy set of a fuzzy set, briefly review the secant method [9] for solving non-linear functions, and propose the definition of piecewise fuzzy entropies between any two characteristic points of fuzzy sets. In Section 3, we propose a new method for weighted fuzzy interpolative reasoning based on the proposed piecewise fuzzy entropies of fuzzy sets. In Section 4, we apply the proposed weighted fuzzy interpolative reasoning method to deal with the multivariate regression problems [29], the Mackey–Glass chaotic time series prediction problem [28] and the time series prediction problems [6,40]. We also make a comparison of the performance of the proposed method with the existing methods [1,7,14,20,38] and [52]. The conclusions are discussed in Section 5.

2. Preliminaries

2.1. Polygonal fuzzy sets

A polygonal fuzzy set A in the universe of discourse X can be characterized by n characteristic points $a_0, a_1, \ldots, a_l, a_r, \ldots, a_{n-2}$ and a_{n-1}, as shown in Fig. 1, where $A = (a_0, a_1, \ldots, a_l, a_r, \ldots, a_{n-2}, a_{n-1}; \mu_0, \mu_1, \ldots, \mu_l, \mu_r, \ldots, \mu_{n-2}, \mu_{n-1})$, the degrees of membership of the characteristic points $a_0, a_1, \ldots, a_l, a_r, \ldots, a_{n-2}$ and a_{n-1} are $\mu_0, \mu_1, \ldots, \mu_l, \mu_r, \ldots, \mu_{n-2}$ and μ_{n-1}, respectively,