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Compressive image sensing for fast recovery from limited samples: A variation on compressive sensing

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A R T I C L E I N F O

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In order to attain better reconstruction quality from compressive sensing (CS) of images, exploitation of the dependency or correlation patterns among the transform coefficients commonly has been employed. In this paper, we study a new image sensing technique, called compressive image sensing (CIS), with computational complexity $O(m^2)$, where m denotes the length of a measurement vector $y = \phi x$, which is sampled from the signal x of length n via the sampling matrix ϕ with dimensionality $m \times n$. CIS is basically a variation on compressive sampling.

The contributions of CIS include: (i) reconstruction speed is extremely fast due to a closed-form solution being derived; (ii) certain reconstruction accuracy is preserved because significant components of x can be reconstructed with higher priority via an elaborately designed ϕ; and (iii) in addition to conventional 1D sensing, we also study 2D separate sensing to enable simultaneous acquisition and compression of large-sized images.

1. Introduction

In this section, we describe the background of compressed sensing in Section 1.1, discuss related work in Section 1.2, and describe the contributions and overview of proposed method in Section 1.3, before providing the outline of this paper in Section 1.4.

1.1. Background on compressed sensing

Compressed/Compressive sensing (CS) has received considerable attention recently due to its revolutionary development in simultaneously sensing and compressing signals with certain sparsity. Moreover, the architecture of the so-called single-pixel camera [19,30] has promoted the practicality of compressed sensing for images. CS is mainly composed of two steps. Let x denote a k-sparse signal of length n to be sensed, let ϕ of dimensionality $m \times n$ represent a sampling matrix, and let y be the measurement of length m. At the encoder, a signal x simultaneously is sensed and compressed via random projection, and the obtained samples are called measurements y in the context of compressed sensing. They are related via random projection as:

$$y = \phi x. \tag{1}$$

The measurement rate is defined as $0 < \frac{m}{n} < 1$ or $0 < \frac{m}{n} << 1$, which indicates the compression ratio (without quantization) without storing the original signal of length n. At the decoder, the original signal x to be sensed can be perfectly recovered by

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means of convex optimization or greedy algorithms if the relationship between m and k, i.e.,

$$m = O\left(k \cdot \log\left(\frac{n}{k}\right)\right), \quad (2)$$

is satisfied [7].

For convex optimization-based CS algorithms, sparse signal recovery will be time-consuming and intractable if ℓ_0-minimization is adopted. ℓ_0-minimization seeks to find k non-zero entries of a signal if the signal is k-sparse in either the time/space or transform (e.g., DCT or wavelet) domain. The solution can become more tractable if the constraint of ℓ_0-minimization is relaxed and ℓ_1-minimization is used instead. Several algorithms relying on ℓ_1-minimization have been presented in the literature.

In addition to convex optimization, non-convex programming (or greedy) algorithms, like Orthogonal Matching Pursuit (OMP) [41], are an alternative for sparse signal recovery. Basically, OMP has been recognized as a "fast" algorithm with time complexity $O(kmn)$ with reasonable reconstruction quality in some cases.

On the other hand, in the context of compressed sensing (CS) [15], the constraint of sparsity enables the possibility of sparse signal recovery to use the measurements with the number (far) fewer than the original signal length. Moreover, the measurements generated from random projection of the original signal via a sampling matrix are weighted equally; i.e., no measurement is more significant than the others. Thus, CS inherently is weakened in handling less-sparse signals, such as highly textured images. The problem here is if we can yield weighted measurements so that less sparse signals can be quickly reconstructed while maintaining good reconstruction quality. Namely, we seek to find approximate reconstruction instead of an exact reconstruct for multimedia data that permit certain content loss.

1.2. Related work: CS methods exploiting known sparsity patterns or partially known support

In the compressed sensing literature, many studies have explored the structure or correlation inherent in the transformed coefficients in order to better reconstruct the signal from its corresponding measurement vector. Inspired by the concept of JPEG2000 compression, the tree structure of wavelet transform has been exploited popularly.

In [16,17], instead of capturing non-adaptive or universal measurements, the authors propose attaining adaptive transform coefficients by exploiting the tree structure of the Haar wavelet. In terms of image quality and recovery speed, the so-called adaptive compressed sensing framework demonstrates its superiority over its non-adaptive counterparts.

In [24], a tree-structured Bayesian compressed sensing framework is proposed, wherein the hierarchical statistical models of wavelet and DCT were adopted and Markov chain Monte Carlo (MCMC) inference was employed. The computationally inefficient MCMC mechanism later is replaced with variational analysis in [25] to speed up recovery. Results show that their methods can achieve both accurate and fast CS recovery. The paradigm in [24,25] belongs to probabilistic structured sparsity [2].

Moreover, the concept of clustered sparsity has received considerable attention in compressed sensing. As summarized in [2] and Table I of [46], many existing CS algorithms [3,11,12,20,21,26,40] exploiting clustered sparsity need to know some pre-defined information, such as numbers, sizes, and positions of clusters, along with the degree of sparsity. In [46], the proposed Bayesian compressed sensing method, a kind of nonparametric recovery algorithm, could make use of clustered sparsity without relying on prior information. Basically, [46] is inspired by [25] in that variational analysis was used in place of MCMC for Bayesian inference in order to guarantee convergence within finite iterations. The major difference between [25] and [46] is that the former employs a directional graphical model for the tree structure of wavelet coefficients, while the latter uses an undirectional graphical model. Furthermore, in order to target the problem of reconstructing structured-sparse signals, belief propagation is employed in [39], which resembles turbo equalization from digital communications. The clustered sparsity-based compressed sensing methods mentioned above belong to deterministic structured sparsity [2].

It should be noted that, in [3], both tree structure and structured sparsity are considered and incorporated into two state-of-the-art CS algorithms, which are CoSaMP [34] and iterative hard thresholding (IHT) [4].

Recently, a so-called N-BOMP (N-way block OMP) method [5] has been developed based on exploiting Kronecker product and block sparsity. The authors prove the equivalence between the Tucker model and Kronecker representation for multiway arrays, thus, Kronecker structure can be used to solve the Tucker model-based underdetermined linear systems within compressive sensing. N-BOMP outperforms the existing tensor-based CS algorithms in that block sparsity of tensor is exploited such that the Kronecker dictionary can be used to speed recovery and improve reconstruction quality. Nevertheless, these advantages come from (also indicated in Subsection 7.2.1 of [5]) the assumption that, for a 2D image, it is pre-processed in advance to possess a perfect block sparsity pattern in that the important/significant coefficients in some transform domains fall within the specified block sparsity pattern while other insignificant coefficients are removed entirely. Therefore, N-BOMP is able to obtain reconstruction quality far better than the existing tensor CS algorithms under the prerequisite/restriction. Later, without making any assumptions about the sparsity pattern, Caiafa and Cichocki [6] present a fast non-iterative tensor compressive sensing method. It, however, assumes that the signal to be sensed and recovered has low multilinear-rank, leading to redundant sensing. This means that, under the same measurement rate, the reconstructed quality is (remarkably) lower than other CS algorithms.

In [29], we propose the use of tree structure sparsity pattern (TSSP) in tensor compressive sensing. TSSP can help to quickly find significant wavelet coefficients and save the execution time to calculate the maximum correlations in greedy algorithms. Its weakness is that there is no fast recovery algorithm that can exploit TSSP.

In addition to the aforementioned sparsity patterns, including the tree structure and clustered/block sparsity, other models of transform coefficients, including Laplacian scale mixtures [8], piecewise autoregressive model [44], Laplace prior [1], and

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