Information Sciences 325 (2015) 159–174

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Generalized type-2 fuzzy weight adjustment for backpropagation neural networks in time series prediction

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ARTICLE INFO

Article history:
Received 27 August 2014
Revised 5 April 2015
Accepted 4 July 2015
Available online 11 July 2015

Keywords:
Neural network
Generalized type-2 fuzzy weight
Backpropagation algorithm
Generalized type-2 fuzzy system

ABSTRACT

In this paper the comparison of a proposed neural network with generalized type-2 fuzzy weights (NNGT2FW) with respect to the monolithic neural network (NN) and the neural network with interval type-2 fuzzy weights (NNIT2FW) is presented. Generalized type-2 fuzzy inference systems are used to obtain the generalized type-2 fuzzy weights and are designed by a strategy of increasing and decreasing an epsilon variable for obtaining the different sizes of the footprint of uncertainty (FOU) for the generalized membership functions. The proposed method is based on recent approaches that handle weight adaptation using type-1 and type-2 fuzzy logic. The approach is applied to the prediction of the Mackey–Glass time series, and results are shown to outperform the results produced by other neural models. Gaussian noise was applied to the test data of the Mackey–Glass time series for finding out which of the presented methods in this paper shows better performance and tolerance to noise.

1. Introduction

One of the most important parts in the structure of the neural network can be considered to be the weights applied to the neurons, because these enable the learning process in the neural network. This part is the main focus of this paper, because the use of generalized type-2 fuzzy weights to obtain the values for the weights in the connections between the input and hidden layers, and the hidden and output layers for a neural network can improve the learning process. The proposed method is compared against a neural network with interval type-2 fuzzy weights using the same architecture and learning algorithms. Noise was also applied to the real data to analyze the performance of the methods under a higher level of uncertainty.

We propose the adaptation of the weights in the backpropagation algorithm for neural networks using generalized type-2 fuzzy inference systems [2]. This approach is different than the ones previously used in the literature, where the adaptation of the weights is made with a momentum variable and adaptive learning rate [4,20], or using triangular or trapezoidal type-1 fuzzy numbers to describe the weights [28,29]. Also, in previous work we presented interval type-2 fuzzy inference with triangular or Gaussians membership functions to obtain the weights for the neurons in the neural network [21,22].

The proposed approach is applied to time series prediction for the Mackey–Glass time series. The objective of applying different forecasting models is obtaining the minimum prediction error for the data of the time series.

The paper is mainly based on comparing the performance for the neural network with generalized type-2 fuzzy weights against a neural network with interval type-2 fuzzy weights. This is an important issue to investigate; because the weights affect the learning process of the neural network and considering uncertainty in the weights of the neural network allow obtaining

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http://dx.doi.org/10.1016/j.ins.2015.07.020
0020-0255/

better results. We performed experiments with the two methods and presented the results and comparison of these methods for the prediction of the Mackey–Glass time series.

The main contribution is the proposed adaptation of the backpropagation algorithm to allow the neural network to manage data with uncertainty, using generalized type-2 fuzzy logic for obtaining generalized type-2 fuzzy weights in the connections between the neurons of the layers. This adaptation of the backpropagation algorithm using generalized type-2 fuzzy inference systems to generate the type-2 weights enables the neural network to handle data with noise, such as that of the Mackey–Glass time series and other complex time series.

The paper is structured as follows: the next section presents a background of different methods for managing the weights and modifications of the backpropagation algorithm in neural networks, and basic concepts of neural networks and generalized and interval type-2 fuzzy logic. Section 3 explains the proposed method and the problem description, the weights using generalized type-2 fuzzy systems, and the neural network architecture with generalized type-2 fuzzy weights proposed in this paper. Section 4 presents the simulation results for the proposed methods. Finally, in Section 5 the conclusions of this work are presented.

2. Background and basic concepts

2.1. Neural networks

An artificial neural network is based on the processing of artificial neurons connections [14,37]. The artificial neuron is composed of several elements as described below.

First we find the inputs and the weights for each input, these are connected to the neuron and then the neuron performs a summation of the multiplication of the input values for the weights ($\sum^{x_i} w_{ij}$), and finally the activation function is used. This function is completed with the addition of a threshold amount *i*. This threshold has the same effect of an input with a value of -1. It serves so that the sum can be shifted to the left or right of the origin [15,53]. After the addition, we have the *f* function applied to the sum, resulting in the final value of the output, also called y_i, and obtaining the following equation:

$$y_i = f\left(\sum_{i=1}^{n} x_i w_{ij}\right) \tag{1}$$

Where *f* may be a nonlinear function with a binary output $+-1$, a linear function $f(z) = z$, or as sigmoid logistic function: $(z) = \frac{1}{1+e^{-z}}$.

2.2. Generalized and interval type-2 fuzzy logic

Fuzzy logic enables a computer system to reason with uncertainty [8]; the fuzzy inference system consists of a set of if–then rules defined over fuzzy sets. A type-2 fuzzy set is characterized by a fuzzy membership function, i.e., the membership degree for each element of this set is a fuzzy set in [0, 1], in contrast to a type-1 fuzzy set, where the membership grade is a real number in [0, 1] [9].

Type-2 fuzzy sets can be used in situations where there is an uncertainty about the membership degrees themselves. We use a type-2 fuzzy set when the transition from ordinary sets to fuzzy sets in any situation is so fuzzy that we have a trouble determining the membership degree even as a real number in [0, 1], and we use type-1 fuzzy sets when we cannot determine the membership of an element in a set as 0 or 1 [10,13].

A generalized type-2 fuzzy set (T2 FS), denoted by $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x, u)$, where $x \in X,\ u \epsilon J_x \subseteq [0, 1]$ and $0 \le \mu_{\tilde{A}}(x, u) \le 1$, and can be represented by [34,43,44,60,61]:

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) | \forall \mathrm{x} \in \mathrm{X}, \forall \mathrm{u} \in \mathrm{J_x} \subseteq [0, 1]\} \tag{2}$$

If $\tilde{A}$ is continuous it can be denoted by:

$$\tilde{A} = \left\{\int_{x\in X} \frac{\mu_{\tilde{A}}(x)}{x}\right\} = \left\{\int_{x\in X}\int_{u\in J^u_{x\subseteq[0,1]}} \frac{\mu_{\tilde{A}}(x, u)}{(x, u)}\right\} = \left\{\frac{\int_{x\in X}\left[\int_{u\in J^u_{x\subseteq[0,1]}} \frac{f_x(x,u)}{u}\right]}{x}\right\} \tag{3}$$

Where $\int\int$ denotes the union for *x* and *u*.Where J_x is called the primary membership of *x* in $\tilde{A}$. At each value of *x* say $x = x'$, the two-dimensional (2-D) plane, whose axes are *u* and $\mu_{\tilde{A}}(x', u)$, is called a vertical slice of $\tilde{A}$ [43]. A secondary membership function is a vertical slice of $\mu_{\tilde{A}}(x, u)$. It is $\mu_{\tilde{A}}(x = x', u)$, for $x' \in X$ and $\forall u \in J_{x'} \subseteq [0, 1]$, and it is described as:

$$\mu_{\tilde{A}}(x = x', u) \equiv \mu_{\tilde{A}}(x', u) = \int_{u\in J_{x'}} \frac{f_{x'}(u)}{u} \tag{4}$$

Where $J_{x'} \subseteq [0, 1]$ and $0 \le f_{x'}(u) \le 1$.