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Single-variable term semi-latticized fuzzy relation geometric programming with max-product operator

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ABSTRACT

Considering the practical application background in peer-to-peer network system, we investigate the single-variable term semi-latticized fuzzy relation geometric programming with max-product operator in this paper. Based on the characteristics of the objective function and feasible domain, we develop a matrix approach to deal with the proposed problem. A step-by-step algorithm is proposed to find an optimal solution without solving all the minimal solutions of the constraint. Two examples are given to illustrate the feasibility and efficiency of the algorithm.

1. Introduction

Since Sanchez [1,2] investigated the fuzzy relation equations with max–min composition operator in 1976 for the first time, many scholars became interested in this research topic and developed various effective resolution methods [4–11,35]. Fuzzy relation equation was applied to medical diagnosis in Brouwerian logic [3], business management [28] and other fields [12–16,40].

A mathematical programming with fuzzy relation equations or inequalities constraint is usually called a fuzzy relation optimization problem. Due to the special structure of the feasible domain, i.e. solution set of a system of fuzzy relation equations or inequalities, the solution method of this kind of problems is much different from that of the ordinary optimization problem. The fuzzy relation optimization problem has been an attractive research topic since Wang et al. [17] investigated the following model:

$$\begin{aligned} \min\ & z = \bigvee_{j=1}^{n} (c_j \wedge x_j) \\ \text{s.t.}\ & B \le A \circ x \le D, \\ & x \in [0,1]^n. \end{aligned} \tag{1}$$

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This is a latticized linear programming problem subject to the fuzzy relation inequalities with max–min composition operator. In [17], the authors used the conservative path method to obtain the minimal solution set of the max–min fuzzy relation inequalities. Based on the minimal solution set, the fuzzy relation latticized linear programming problem was solved. Recently the latticized linear programming problem was studied in [38,39], and the solution method proposed by Wang et al. was improved.

Some researchers focused on the fuzzy relation linear programming problem [18–23,41]. Loetamonphong and Fang [18] studied the linear programming problem subject to max-product fuzzy relation equations. The main problem was separated into two sub-problems. One of the sub-problems could be easily solved and the other was converted into a 0–1 integer programming and solved by the branch-and-bound method. In [18], without finding all the minimal solutions, the authors searched the optimal solution from a solution set which lied between the set of all minimal solutions and the set of all feasible solutions.

In recent years, the fuzzy relation nonlinear programming problem has been studied. In [24–27], the authors applied the genetic algorithm to optimize the general nonlinear objective function subject to fuzzy relation equations or inequalities. The genetic algorithm is an effective solution method. However, the authors can only obtained the approximate optimal solution, but not the exact optimal solution of the problem.

Cao was the pioneer in the research topic of fuzzy geometric programming problem [36,37]. Yang and Cao studied the fuzzy relation geometric programming problem for the first time [30,31]. In [31], Yang and Cao introduced the monomial geometric programming with max–min fuzzy relation equation constraints. The main problem was separated into two simple subproblems. One of the subproblems had non-negative exponents in the objective functions, and the other one had negative ones. After finding out the maximum solution and all the minimal solutions of the max–min fuzzy relation equations, two subproblems were solved respectively. And then the optimal solution of the main problem may be easily obtained.

Considering the monomial geometric programming with fuzzy relation inequality constraints with max-product composition, Shivanian and Khorram [32] improved the resolution method in [31]. Simplification operations have been given to accelerate resolution of the problem by removing the components having no effect on the solution process [32].

Wu [33] optimized the geometric programming problem with single-term exponents subject to max–min fuzzy relational equation constraints as follows:

$$\begin{aligned} &\min z = \bigvee_{i=1}^{m} (c_i \wedge x_i^{r_i}) \\ &\text{s.t.}\, x \circ A = b, \\ &x \in [0,1]^m. \end{aligned} \tag{2}$$

Instead of general arithmetic operations, the objective function in this model was formulated by the lattice operators $\vee$ and $\wedge$. Problem (2) may be called latticized fuzzy relation geometric programming problem with single-term exponents. Using the simple value matrix, Wu obtained an optimal solution of problem (2) without looking for all the potential minimal solutions.

X.G. Zhou and R. Ahat [34] investigated the following problem:

$$\begin{aligned} &\min z = \bigvee_{i=1}^{m} (c_i \cdot x_i^{r_i}) \\ &\text{s.t.}\, x \circ A = b, \\ &x \in [0,1]^m. \end{aligned} \tag{3}$$

This is a semi-latticized fuzzy relation geometric programming problem with single-term exponents since the objective function contains the general multiple operation $\cdot$ and the lattice operator $\vee$. Similar to the method presented in [33], the main problem was converted into two subproblems with simple objective functions. However, Zhou and Ahat obtained an optimal solution without using the value matrix. One of the subproblems may be solved easily. And they developed an effective procedure to search an optimal solutions of the other one from a subset of the feasible region.

A system of fuzzy relation equations with max-product composition operator (max-product fuzzy relation equations) is described as:

$$A \circ x = b, \tag{4}$$

where $A = (a_{ij})_{m\times n}$, $x = (x_1, x_2, \ldots, x_n)^T$, $b = (b_1, b_2, \ldots, b_m)^T$, $a_{ij}, x_j, b_i \in [0,1]$, $i \in I = \{1, 2, \ldots, m\}$, $j \in J = \{1, 2, \ldots, n\}$, and $\circ$ denotes the max-product operator. Here, I and J are two index sets.

In this paper, as an extension, we introduce the following single-variable term semi-latticized fuzzy relation geometric programming problem:

$$\begin{aligned} &\min z(x) = \bigvee_{j=1}^{n} f_j(x_j) = \bigvee_{j=1}^{n} \left(\bigvee_{k=1}^{p_j} (c_{jk} \cdot x_j^{\gamma_{jk}}) \right) \\ &\text{s.t.}\, A \circ x = b, \end{aligned} \tag{5}$$

where $f_j(x_j) = \bigvee_{k=1}^{p_j} (c_{jk} \cdot x_j^{\gamma_{jk}})$, $c_{jk} > 0$, $\gamma_{jk} \neq 0$, $\forall k \in \{1, 2, \ldots, p_j\}$, $j \in J$, and $A \circ x = b$ is a system of max-product fuzzy relation equations with $b > (0, 0, \ldots, 0)^T$ (see Definition 1 and the application background of problem (5) in Section 2).