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Ordering finitely generated sets and finite interval-valued hesitant fuzzy sets

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ABSTRACT

Ordering sets is a long-standing open problem due to its remarkable importance in many areas such as decision making, image processing or human reliability. This work is focused on introducing methods for ordering finitely generated sets as a generalization of those methods previously defined for ordering intervals. In addition, these orders between finitely generated sets are also improved to present orders between finite interval-valued hesitant fuzzy sets. Finally, finite interval-valued hesitant fuzzy preference relations are introduced and used to define a new order between finite interval-valued hesitant fuzzy sets.

1. Introduction

Since, Zadeh introduced fuzzy sets to model the uncertainty associated to the concept of imprecision ([36]), several extensions of fuzzy sets have been introduced: interval-valued fuzzy sets ([13,27,37]), Atanassov's intuitionistic fuzzy sets ([1,2]), hesitant fuzzy sets ([18,25,30]), typical hesitant fuzzy sets ([5]), fuzzy sets of type 2 ([38]), etc. In particular, interval-valued fuzzy sets have been deeply studied. For example Zhang et al. develop in [40] an adjustable approach to interval-valued intuitionistic fuzzy soft sets and define the concept of weighted interval-valued intuitionistic fuzzy soft set. In [24] the problem of the interval-valued fuzzy sets synthesis is studied. Q2

All these extensions have received an increasing interest in different fields such as classification ([14,28]), human reliability ([23]), image processing ([7]). In particular, there are a lot of works focused on solving decision making problems using extensions of fuzzy sets. We highlight the works developed in [32], where the aggregation of hesitant fuzzy information is studied. In addition, in [39] new aggregation operators are utilized to develop techniques for multiple attribute group decision making with hesitant fuzzy information. Liu and Sun in [21] develop a generalized power hesitant fuzzy ordered weighted average operator to aggregate hesitant fuzzy numbers. Finally, Chen et al. in [11] introduce interval-valued hesitant preference relations to describe uncertain evaluation information in group decision making processes.

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Therefore, an order between the objects of these extensions is necessary to properly implement these applications. Several orders have been studied and defined between fuzzy sets during the last 20 years ([10,12,20,22,35]).

Regarding ordering for interval-valued fuzzy sets, Barrenechea et al. in [3] develop a construction method for interval-valued fuzzy preference relations from a fuzzy preference relation. They represent the lack of knowledge or ignorance that experts suffer when they define the membership values of the elements of that fuzzy preference relation. In addition, they propose a generalization of Orlovsky's non dominance method to solve decision making problems using interval-valued fuzzy preference relations. Bustince et al. in [9] address the problem of choosing a total order between intervals. Their procedure is based on studying firstly the additivity of interval-valued aggregation functions. Then, they treat the problem of preserving admissible orders by linear transformations. Finally, they study the construction and properties of interval-valued ordered weighted aggregation operators by means of admissible orders.

Nevertheless, despite this kind of sets are reaching the spotlight in recent years, orders between hesitant fuzzy sets (and their extensions) have not been deeply explored yet. Actually, to the best of our knowledge, literature about orders between hesitant fuzzy sets (and their extensions) is sparse, even if there exists some works which have dealt with orders for typical hesitant fuzzy-sets ([4,5]).

The goal of this paper is twofold. Firstly, finite interval-valued fuzzy sets, which are a natural extension of typical hesitant fuzzy sets, are introduced. This kind of sets could immediately derive on lots of applications in several fields such as, for example, group decision making. For instance, this new kind of sets could model, at the same time, experts and criteria.

Furthermore, as those new sets are defined by a membership function which is the union of disjoint closed intervals (finitely generated sets), the study of these finitely generated sets turns into one of the key points of this paper. Therefore, this kind of sets is deeply analyzed and the concept of α^{sg}-point is introduced. This α^{sg}-point measures, according to a parameter, the degree of optimism adopted when comparing finitely generated sets.

On the other hand, when using finite interval-valued hesitant fuzzy sets in group decision making, we will need to define a way of comparing these sets. Hence, several orders between finite interval-valued hesitant fuzzy sets are introduced in the last sections of this paper. To that end, α^{sg}-projections and finite interval-valued hesitant fuzzy preference relations are introduced.

The structure of this paper is as follows: Section 2 gives an overview of the preliminary definitions used in this paper. In addition, some methods for ordering real intervals are also reviewed in order to improve them for ordering finitely generated sets in Section 3. Section 4 is devoted to construct several orders between finite interval-valued hesitant fuzzy sets. Finally, some conclusions and open problems are analyzed in Section 5.

2. Preliminary definitions

This section is devoted to briefly introduce several well-known basic concepts and to fix the notations used in this paper.

2.1. Fuzzy sets and their extensions

Definition 1. ([36]) A fuzzy set A over X is an object:

$$A = \{(x, \mu_A(x)) | x \in X\},$$

where $\mu_A: X \to [0, 1]$ is called membership function.

The set of all ordinary fuzzy sets that can be defined on the universe [0, 1] is denoted by $F([0, 1])$.

By abuse of notation, in the literature the membership function is frequently denoted by A insted of μ_A.

In some cases, the uncertainty measured by the fuzzy sets is not enough or it does not fit with the nature of the problem. In other cases, it is not possible to find an accurate way to define the membership functions. In these cases, it is common to make use of the so-called extensions of fuzzy sets. The most relevant ones are defined below.

Definition 2. ([1]) An Atanassov's intuitionistic fuzzy set A on the universe X is defined as

$$A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\},$$

where $\mu_A, \nu_A: X \to [0, 1]$ satisfy

$$\mu_A(x) + \nu_A(x) \le 1, \ \forall x \in X.$$

Here μ_A and ν_A define, respectively, the degree of membership and the degree of non-membership of the element x to the set A.

Definition 3. ([27]) An interval-valued fuzzy set A on the universe X is defined by a mapping

$$A : X \to L([0, 1]),$$

such that the membership degree of $x \in X$ is given by $A(x) = [\underline{A}(x), \overline{A}(x)] \in L([0, 1])$, where $\underline{A} : X \to [0, 1]$ and $\overline{A} : X \to [0, 1]$ are, respectively, mappings defining the lower and the upper bound of the membership interval $A(x)$ and $L([0, 1])$ denotes the set of all closed subintervals in [0, 1]. The class of all interval-valued fuzzy sets on X is denoted by $IVFS(X)$.

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