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Fuzzy interpolative reasoning based on ranking values of polygonal fuzzy sets and automatically generated weights of fuzzy rules

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ABSTRACT

In this paper, we propose a new fuzzy interpolative reasoning method for sparse fuzzy rule-based systems based on ranking values of polygonal fuzzy sets and automatically generated weights of fuzzy rules. First, the proposed method gets the characteristic points of the observation polygonal fuzzy sets and the characteristic points of the antecedent polygonal fuzzy sets of the fuzzy rules, respectively, by using the α-cut operations, respectively, where $\alpha \in [0, 1]$. Then, it calculates the ranking value of each observation polygonal fuzzy set and the ranking value of each antecedent polygonal fuzzy set, respectively. Then, it calculates the difference between the ranking value of each observation polygonal fuzzy set and the ranking value of each antecedent polygonal fuzzy set. Then, it calculates the weight of each fuzzy rule based on the obtained differences between the ranking values of the observation polygonal fuzzy sets and the ranking values of the antecedent polygonal fuzzy sets. Then, based on the obtained characteristic points of the observation polygonal fuzzy sets, the obtained characteristic points of the antecedent polygonal fuzzy sets, and the obtained weights of the fuzzy rules, it gets the characteristic points of the fuzzy interpolative reasoning result represented by a polygonal fuzzy set. The experimental results show that the proposed method can overcome the drawbacks of the existing methods for fuzzy interpolative reasoning in sparse fuzzy rule-based systems.

1. Introduction

Fuzzy interpolative reasoning is a very important research topic for sparse fuzzy rule-based systems. In recent years, some fuzzy interpolative reasoning methods [1–27,29–34,36–46] for sparse fuzzy rule-based systems have been presented. In [3], Chang et al. pointed out some drawbacks of the existing fuzzy interpolative reasoning methods [3,19–21,29], shown as follows:

(1) The fuzzy interpolative reasoning methods presented in [20] and [29] have the drawback that they cannot preserve the convexity of the fuzzy interpolative reasoning result.
(2) The drawback of the fuzzy interpolative reasoning method presented in [19] is that it only can deal with fuzzy interpolative reasoning using triangular membership functions; the drawback of the fuzzy interpolative reasoning method presented

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in [20] is that it only can deal with fuzzy interpolative reasoning using triangular membership functions and trapezoidal membership functions.

(3) The fuzzy interpolative reasoning method presented in [20] has the drawback that they cannot deal with fuzzy interpolative reasoning with multiple antecedent variables.

(4) The fuzzy interpolative reasoning method presented in [3] and [21] have the drawback that they may generate the same fuzzy interpolative reasoning result with respect to different observations by using representative values of fuzzy sets.

Moreover, in this paper, we also find that Chen et al.'s method [9] gets unreasonable fuzzy interpolative reasoning results in some situations and Yang and Shen's method [46] has the following drawbacks:

(1) If the observation fuzzy sets are crisp values, then it cannot get a crisp-valued fuzzy interpolative result, which is not reasonable for fuzzy interpolative reasoning in sparse fuzzy rule-based systems.

(2) It gets unreasonable fuzzy interpolative results in some situations in terms of the shapes of the membership functions of the fuzzy interpolative results.

Therefore, we need to develop a new fuzzy interpolative reasoning method to overcome the drawbacks of the methods presented in [3,9,19–21,29,46].

In this paper, we propose a new fuzzy interpolative reasoning method for sparse fuzzy rule-based systems based on ranking values of polygonal fuzzy sets and automatically generated weights of fuzzy rules. First, the proposed method gets the characteristic points $a^*_{k,0}, a^*_{k,1}, \ldots, a^*_{k,s-2}$ and $a^*_{k,s-1}$ of the observation polygonal fuzzy sets A^*_k and gets the characteristic points $a_{ik,0}, a_{ik,1}, \ldots, a_{ik,s-2}$ and $a_{ik,s-1}$ of the antecedent polygonal fuzzy set A_{ik} of fuzzy rule R_i by the α-cuts of polygonal fuzzy sets, respectively, where $\alpha \in [0, 1]$, $1 \le i \le n$, $1 \le k \le m$ and s is the number of the characteristic points of the polygonal fuzzy set. Then, it calculates the ranking value $Rank(A^*_k)$ of observation polygonal fuzzy set A^*_k and calculates the ranking value $Rank(A_{ik})$ of antecedent polygonal fuzzy set A_{ik}, respectively, where $1 \le i \le n$ and $1 \le k \le m$. Then, it calculates the difference $D((Rank(A^*_k), Rank(A_{ik}))$ between the ranking value $Rank(A^*_k)$ of observation polygonal fuzzy set A^*_k and the ranking value $Rank(A_{ik})$ of antecedent polygonal fuzzy set A_{ik}, where $1 \le i \le n$ and $1 \le k \le m$. Then, it calculates the weight W_i of fuzzy rule R_i based on the obtained difference $D(Rank(A^*_k), Rank(A_{ik}))$ between the ranking value $Rank(A^*_k)$ of observation polygonal fuzzy set A^*_k and the ranking value $Rank(A_{ik})$ of antecedent polygonal fuzzy set A_{ik}, where $1 \le i \le n$. Finally, based on the obtained characteristic points $a^*_{k,0}, a^*_{k,1}, \ldots, a^*_{k,s-2}$ and $a^*_{k,s-1}$ of the observation polygonal fuzzy sets A^*_k, the obtained characteristic points $a_{ik,0}, a_{ik,1}, \ldots, a_{ik,s-2}$ and $a_{ik,s-1}$ of the antecedent polygonal fuzzy sets A_{ik}, and the obtained weight W_i of each fuzzy rule R_i, it gets the characteristic points $b^*_0, b^*_1, \ldots, b^*_{s-2}$ and b^*_{s-1} of the fuzzy interpolative reasoning result B^* represented by a polygonal fuzzy set, respectively, where $1 \le i \le n$, $1 \le k \le m$ and s is the number of the characteristic points of the polygonal fuzzy set.

The main contribution of this paper is that we propose a new fuzzy interpolative reasoning method for sparse fuzzy rule-based systems based on ranking values of polygonal fuzzy sets and automatically generated weights of fuzzy rules. The experimental results show that the proposed fuzzy interpolative reasoning method can overcome the drawbacks of the methods presented in [3,9,19–21,29,46].

The rest of this paper is organized as follows. In Section 2, we briefly review the definitions of fuzzy sets [47], α-cuts [28] of fuzzy sets and polygonal fuzzy sets, where $\alpha \in [0, 1]$. In Section 3, we briefly review the definition of ranking values [15] of polygonal fuzzy sets. In Section 4, we propose a new fuzzy interpolative reasoning method for sparse fuzzy rule-based systems based on ranking values of polygonal fuzzy sets and automatically generated weights of fuzzy rules. In Section 5, we use some examples to compare the fuzzy interpolative reasoning results of the proposed method with the ones of the existing methods [3,9,19–21,29,46]. The conclusions are discussed in Section 6.

2. Preliminaries

In this section, we briefly review the definitions of fuzzy sets [47], α-cuts [28] of fuzzy sets, where $\alpha \in [0, 1]$, and polygonal fuzzy sets.

The fuzzy set theory [47] was proposed by Zadeh in 1965. A fuzzy set A in the universe of discourse U, $U = \{x_1, x_2, \ldots, x_n\}$, is characterized by a membership function μ_A represented as follows:

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \cdots + \mu_A(x_n)/x_n, \tag{1}$$

where $\mu_A(x_i)$ denotes the membership degree of element x_i belonging to the fuzzy set A, $\mu_A(x_i) \in [0, 1]$ and $1 \le i \le n$. If an element x_k exists, such that $\mu_A(x_k) = 1$, where $x_k \in U$, then the fuzzy set A is called a normal fuzzy set [14]. For all elements x_1 and x_2 in the universe of discourse U, if the following condition holds, then the fuzzy set A is called a convex fuzzy set [14]:

$$\mu_A(\lambda x_1 + (1 - \lambda)x_2) \ge \mathrm{Min}\{\ \mu_A(x_1),\ \ \mu_A(x_2)\}, \tag{2}$$

where $\mu_A(x_1)$ and $\mu_A(x_2)$ denotes the membership degrees of the elements x_1 and x_2 belonging to the fuzzy set A, respectively, and $\lambda \in [0, 1]$.

Definition 2.1 [47]**.** Let A and B be two fuzzy sets in the universe of discourse U. The intersection between the fuzzy sets A and B is defined as follows:

$$\mu_{A \cap B}(x) = \min\{\mu_A(x),\ \mu_B(x)\}, \quad \forall x \in U, \tag{3}$$