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Decision-theoretic rough fuzzy set model and application

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ABSTRACT

This article investigates the rough approximation of a fuzzy concept on a probabilistic approximation space. We propose the probabilistic rough fuzzy set by defining the conditional probability of a fuzzy event. Then we establish the model of probabilistic rough fuzzy set and discuss several properties in detail. Furthermore, three generalizations of probabilistic rough fuzzy set, namely, 0.5-probabilistic rough fuzzy set, variable precision probabilistic rough fuzzy set and Bayesian rough fuzzy set are reported. In order to give a systematic method of selecting parameters for the probabilistic rough fuzzy set, we propose a decision-theoretic rough fuzzy set. That is, we formulate a non-parametric definition of the probabilistic rough fuzzy set. Moreover, we illustrate the motivation and verify the validity of the decision-theoretic rough fuzzy set by using a credit card applicant decision-making problem. Furthermore, the interrelationship between the decision-theoretic rough fuzzy set and the probabilistic rough fuzzy set is explained. The main contribution of this paper is twofold. One is to extend the probabilistic rough set to fuzzy environment, i.e., the probabilistic rough fuzzy set model. Another is to present an approach to select parameters needed in probabilistic rough fuzzy set modeling by using the process of decision-making under conditions of risk.

1. Introduction

Rough set theory is an extension of set theory for studying intelligent systems characterized by insufficient and incomplete information [34,36,37,52]. As a new mathematical tool to deal with vagueness and uncertainty, one of the main advantages of rough set theory is that it does not need any preliminary or additional information about data, such as probability distribution in statistics, basic probability assignment in the Dempster–Shafer theory, or grade of membership or the value of possibility in fuzzy set theory [55]. This new mathematical approach to imprecision, vagueness and uncertainty is founded on the assumption that objects in the universe of discourse can be associated with data or knowledge [31,42,43]. Recently, rough set theory has become an important method and tool for granular computing [34–37,49,50,52] and also has been demonstrated to be useful in decision-making [26,51,53], feature selection [48,72,74,75], clustering analysis [54], machine learning [3,12] and so on.

The theory of rough set was proposed in 1982. It has captured much attention from artificial intelligence and intelligent systems researchers and has been applied to knowledge discovery, data mining, etc. Many generalized rough set models have been proposed by scholars in past years. Broadly speaking, there are several directions for the generalization of the

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rough set theory. In [65], Yao summarizes various formulations of the standard rough set theory and demonstrates how those formulations can be adopted to develop different generalized rough set theories. Within the set-theoretic framework, generalizations of the element-based definition can be obtained by using non-equivalence binary relations [33,63], generalizations of the granule-based definition can be obtained by using coverings [47,48,64,65], and generalizations of the subsystem-based definition can be obtained by using other subsystems [65]. By the fact that the system $(2^U, c, \cap, \cup)$ is based on Boolean algebra, one can generalize rough set theory using other algebraic systems such as Boolean algebras, lattices, and posets [10,35]. Subsystem-based definitions and algebraic methods are useful for such generalizations. Also, there are many other important generalizations of the theory, such as probabilistic and decision-theoretic rough sets [45,55,56] and rough membership functions [31,44,50].

However, many generalized rough set models are often too strict when including objects into the approximation regions and may require additional information. A lack of consideration for the degree of overlap between an equivalence class and the set to be approximated unnecessarily limits the applications of rough set and has motivated a good deal of research to investigate probabilistic generalization of the theory [4,7,28,29,41,45,46,55,59,71]. In 1987, Wong and Ziarko [45] introduced probabilistic approximation space to the studies of rough set and then presented the concept of probabilistic rough set. Subsequently, Yao et al. [66] proposed a more general probabilistic rough set called decision-theoretic rough set. Then another perspective to deal with the degree of overlap of an equivalence class with the set to be approximated was given, and an approach to select the needed parameters in lower and upper approximations was presented. As far as the probabilistic approach to rough set theory, Pawlak and Skowron [31], Pawlak et al. [32] and Wong and Ziarko [45] proposed a method to characterize a rough set by a single membership function. By the definition of a rough membership function, elements in the same equivalence class have the same degree of membership. The rough membership may be interpreted as the probability of any element belonging to a set, given that the element belongs to an equivalence class. This interpretation leads to probabilistic rough set [56]. Greco et al. [6] introduced a new generalization of the original definition of rough sets and variable precision rough sets, named the parameterized rough set model. They aim at modeling data relationships expressed in terms of frequency distribution rather than in terms of a full inclusion relation, which is used in the classical definition of rough sets.

Probabilistic rough set extends the classical Pawlak rough set model. The major change is the consideration regarding the probability of an element being in a set to determine inclusion in approximation regions. Two probabilistic thresholds are used to determine the division between the boundary-positive region and boundary-negative region. Over the last two decades, probabilistic rough set theories, such as 0.5-probabilistic rough set [45], decision-theoretic rough set [57], rough membership function [31], parameterized rough set [38], Bayesian rough set [39], game-theoretic rough set [8] and naive Bayesian rough set [39,60] have been proposed to solve probabilistic decision-making problems by allowing a certain acceptable level of error. All these models use two parameters α and β (a pair of thresholds) to define the lower and upper approximations. Actually, variable precision rough set [69] proposed by Ziarko also is one kind of probabilistic rough set; he firstly used a set inclusion function to define the lower and upper approximations. Moreover, only one parameter was used in the lower and upper approximations. Later on, he reformulated the theory of variable precision rough set by using probabilistic terms [69]. However, a fundamental difficulty with all generalizations of probabilistic rough set is the physical interpretation of the required threshold parameters [58], as well as the need for a systematic method for setting the parameters. In order to offer an effective approach for selecting the threshold parameters, Yao et al. [57] proposed the decision-theoretic rough set model by using the Bayesian decision theory, and then the above difficulty was resolved from the semantic point of view.

Decision-theoretic rough set, as a general probabilistic rough set model, has invoked the interest of many scholars and much valuable research has been done in recent years. We briefly review the studies of decision-theoretic rough set as follows: Herbert and Yao [8,9] study the combination of the decision-theoretic rough set and the game rough set. Li and Zhou [14,15] present a multi-perspective explanation of the decision-theoretic rough set and discuss attribute reduction and its application for the decision-theoretic rough set. Jia et al. [11] also discuss the attribute reduction problem for the decision-theoretic rough set theory. Liu et al. [16–19] discuss multiple-category classification with decision-theoretic rough sets and its applications in the areas of management science. Li et al. [20], Lingras et al. [21–23] and Yu et al. [68] discuss the clustering analysis by using the decision-theoretic rough set theory. Yang [67] studies the multi-agent decision-theoretic rough set model. Li et al. and Liang et al. [24,25,27] discuss information retrieval and filtering by using the decision-theoretic rough set theory. Greco and Slowinski [5] combine the decision-theoretic rough set with the dominance-based rough set and then give a new generalized rough set model. Based on the basic idea of the decision-theoretic rough set model, Zhou [73] presents a new description of this model. Ma and Sun [28,29] study the decision-theoretic rough set theory over two universes based on the idea of the classical decision-theoretic rough set.

In general, the objects approximated are crisp set or accurate concepts of the universe of discourse in both Yao's decision-theoretic rough set theory and the existing probabilistic rough set models. For a decision-making problem, there are only two states, which are disjoint and opposite each other for a precise concept of the universe of discourse. For example, in the decision-making problems of diagnosis analysis and email spam filtering, there are only two states of *Yes* or *No* for a sufferer or an email. That is, a patient either has the disease or does not have the disease and an email either is junk mail or is not junk mail.

However, the objects of many decision-making problems, such as measuring student achievement in comprehensive testing or the credit evaluation of a credit card applicant, could have more than two states in practice. Moreover, the states of the decision object are not necessarily disjoint and opposite each other. For a given student or credit card applicant, the evaluation results may not be described by two completely opposite states with *Yes* or *No*. That would be the case if a student