Information Sciences 282 (2014) 38–56

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Emergence of universal global behavior from reversible local transitions in asynchronous systems

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ARTICLE INFO

Article history:
Received 13 July 2013
Received in revised form 14 May 2014
Accepted 20 May 2014
Available online 11 June 2014

Keywords:
Local reversibility
Global reversibility
Asynchronous circuit
Cellular automaton
Concurrency
Intrinsic universality

ABSTRACT

Reversible computing usually focuses on how to establish a valid equivalence between the global reversibility and local reversibility in computational systems. Hitherto the equivalence has been precisely developed in cellular automata, combinational circuits and quantum computers, but it implicitly assumes that the underlying systems are synchronously timed. Alternative systems include the delay-insensitive (DI) circuits and asynchronous cellular automata (ACAs), where the local operations of each component (cell) may be executed independently at random times. Despite the randomness associated with asynchronous timings, equivalence between the global and local reversibility can be simply achieved in both DI-circuits (Morita, 2001) and ACAs (Lee et al., 2003), provided that their local operations (transitions) are thoroughly serialized. The complete exclusion of concurrency in local behavior, however, will profoundly depress the parallel processing efficiency of DI-circuits as well as the intrinsic massive parallelism of ACAs. This paper aims at exploring what kind of complex global behavior may arise from the concurrent operations that are invertible at local level. To this end, we show that DI-circuits composed of reversible elements can actually exhibit universal input and output behavior, with the universality emerging from the concurrency in reversible local operations. Likewise, by further embedding all circuits into the cellular space, the emergence of universal global transitions from reversible local transitions can be exactly identified in an ACA which, due to the bijectivity of its local function, has significantly lower complexity as compared to other models.

1. Introduction

Local reversibility corresponds to the time-reversible nature of microscopic dynamics of particles and fields. Physics at macro-scale to the contrary, is likely to exhibit non-reversibility as described by the Second Law of Thermodynamics. Known as the Loschmidt's Paradox [10], the emergence of macroscopic non-reversible behavior from the microscopic reversible dynamics has been investigated recently in the fields of shockwave [11] and material rheology [43]. For modeling physical systems, *cellular automata* (CAs) are useful tools to explore how and what kind of complex global behavior can probably emerge from the localized interactions among individual cells. They also provide a convenient way to capture the microscopic reversibility in physics in terms of a bijective local transition function [15,31]. Perhaps analogous to the

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http://dx.doi.org/10.1016/j.ins.2014.05.043
0020-0255/

Loschmidt's Paradox, the bijectivity of local transitions in CAs, in general, is unable to decide whether any arbitrary cellular automaton exhibits reversibility in global transitions on its entire cellular space, i.e., the problem is undecidable [14].

Nevertheless, logic reversibility potentially promises to develop computers with radically reduced power consumptions [18,1,5], and is also a precondition for quantum computation [4]. For this reason, extensive efforts have been made to facilitate the constructions of reversible computing models, such as reversible cellular automata [15,31,41,42], reversible logic circuits [6,8,27,44], billiard-ball models [26,3] and quantum computers [4,13,17]. For example, a partitioned cellular automaton (PCA) is a CA where each cell is partitioned into sub-cells in one-to-one correspondence with its neighbors, and each transition rule takes the same number of sub-cells between its left-hand and right-hand sides. The local function's symmetry plays a key role in establishing an equivalence between the global reversibility and local reversibility in PCAs [31], whereby the invertibility of a PCA's local function can automatically give rise to the reversibility in global transitions of the PCA, and vice versa.

As the reversibility of the above models (e.g., a reversible PCA or logic circuit) is usually connected with determinism, their dynamical behavior can be described as a linear graph without branching, in which each vertex represents a global state or an instantaneous configuration of a model, and each arrow pointing from a vertex to another expresses a global transition via which the former configuration can be transformed to the latter. Moreover, a valid equivalence between the global reversibility and local reversibility not only provides an efficient way to test for the reversibility of a computational model, but also can substantially reduce the construction of the inverse of a model to direct reversing the model's local function that is invertible. The inverse of a reversible model is also reversible, which can exactly exhibit the time-reversed dynamics of the original model.

However, the above equivalence as well as the linearity in the graph of all configurations implicitly assume that the underlying reversible model is synchronously timed, i.e., each global transition comprises the local transitions of all elements that are performed in lock step in accordance with a central clock signal. Removal of central clocks may lead to models that can work in asynchronous timings. Delay-insensitive (DI) circuits are one of the most typical examples of asynchronous models, in which signals may be subject to arbitrary delays but without violating the external input and output behavior of the circuits, thereby no need a central clock to synchronize the local operations of circuit elements or transmission of signals. Another well-known example is the asynchronous cellular automata (ACAs) which allow cells to undergo the state transitions at random timings independent from each other.

Because most physical processes at the micro-scale are asynchronous, it makes sense to investigate the reversibility at local and global levels in asynchronous models. Reversible communicating systems [2] attach a memory to each process in a CCS [28] to monitor past behavior along with the progress of the process, which enables backtrack along any casually equivalent past but not the exact reverse of the forward computation. Likewise, local reversibility has been well defined on a special type of logic elements in DI-circuits [30,20,21], each of which takes the same number of input and output lines. Analogous to the reversible logic gates, the symmetry in the inputs and outputs of logic elements allows an effective definition of the bijectivity on their local operations. In spite of the asynchronous timings, local reversibility in logic elements still can directly lead to the global reversibility in a DI-circuit composed of such elements, provided that both the circuit as well as all elements process at most one signal at any time [30,20,23], i.e., they are serial.[1] Because of no concurrent operation, every serial DI-circuit composed of serial reversible elements exhibits linearity in its graph of all configurations, though each transition between a pair of configurations may be subject to arbitrary delays.

Despite the strictly serialized operations, reversible elements are capable of constructing any reversible Turing machine [23], thereby proving their computational (Turing) universality. The reversible Turing machines can be further embedded into a self-timed cellular automaton (STCA [34]), a special type of ACAs, based on a simple set of transition rules that are locally invertible [19]. In particular, the embedding of Turing machines makes at most one cell among all cells be able to undergo state transitions at any time, and hence, no concurrent transition occurs. As with the above serial DI-circuits, the complete exclusion of concurrency in local transitions induces a straightforward equivalence between the global reversibility and local reversibility of the STCA [19], but the dark side of the coin is the loss of the intrinsic capability of CAs to carry out massively parallel computation.

Consider a more general and more practical situation where reversible elements as well as the circuits composed of them enable more than one signal to be processed simultaneously in parallel. In such a case, the concurrency among transmission of multiple signals tends to cause forking and merging of paths in the graph of all possible configurations of a DI-circuit, whereby the global behavior of the circuit might be regarded as incompatible with the reversibility of conventional logic circuits. However, since most branching in the graph results from the concurrent transmission of signals, and the difference in the arrival orders or arrival timings of signals never affects the correct operations of DI-circuits, it is yet unexplored what kind of external input and output behavior the DI-circuits built out of reversible elements can actually exhibit.

This paper aims to explore the constructibility and computability of locally reversible elements for DI-circuits. To this end, we present several novel types of invertible DI elements and show that these elements are capable of constructing any arbitrary DI-circuit, thereby proving the universal constructibility of locally reversible elements. As microscopic physical interactions are inherently asynchronous, the emergence of universal global behavior from reversible local operations in

[1] Thus, at any time, there is at most one element that is activated to process the signal arriving on one of its input lines and transfer the signal to one of its output lines, whereas all other elements in the whole circuit keep inactive until one of them receives the signal from its input lines.