Information Sciences 282 (2014) 57–69

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Fault-tolerant cycles embedding in hypercubes with faulty edges

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ARTICLE INFO

Article history:
Received 12 December 2013
Received in revised form 8 April 2014
Accepted 26 May 2014
Available online 11 June 2014

Keywords:
Hypercube
Cycle embedding
Bipancyclic
Conditional faulty
Fault tolerance
Interconnection network

ABSTRACT

Let Q_n be an n-dimensional hypercube with $f_e \leqslant 3n-8$ faulty edges and $n \geqslant 5$. In this paper, we consider the faulty hypercube under the following two additional conditions: (1) each vertex is incident to at least two fault-free edges, and (2) every 4-cycle does not have any pair of non-adjacent vertices whose degrees are both two after removing the faulty edges. We prove that there exists a fault-free cycle of every even length from 4 to 2^n in Q_n. Our result improves the result by Liu and Wang (2014) in terms of the lengths of embedding cycles, where under the same conditions, a fault-free Hamiltonian cycle was constructed.

1. Introduction

The interconnection network topology is usually represented by a graph, where the vertices represent processors and the edges represent links between processors. Many interconnection network topologies have been proposed for the purpose of connecting hundreds of processing elements. Network architecture design is important in cloud data centers networks. Hua et al. [17] presented the design and implementation of ANTELOPE, a novel datacentric network scheme, for large-scale data centers. Wu et al. [30] proposed a novel cube structure, ARCube (Aggregate-Ranking Cube), for supporting efficient ranking aggregate query processing. Ding et al. [10] studied the problem of keyword search in a data cube with text-rich dimension(s) (so-called *text cube*). Their goal was to find the top-k most relevant cells and they proposed four approaches: *inverted-index one-scan*, *document sorted-scan*, *bottom-up dynamic programming* and *search-space ordering*.

The hypercube is considered to be one of the most versatile and efficient architecture yet discovered for building massively parallel or distributed systems. It possesses many excellent properties such as recursive structure, regularity, symmetry, small diameter, relatively short mean internode distance, low degree, and much small link complexity, which are very important for designing massively parallel for distributed systems [1]. The n-dimensional hypercube [24], denoted by Q_n, is a simple undirected graph with 2^n vertices and $n \cdot 2^{n-1}$ edges. The vertex set is $V(Q_n) = \{u_0u_1 \cdots u_{n-1} \mid u_i \in \{0,1\}$ for $0 \leqslant i \leqslant n-1\}$. Two vertices are adjacent if and only if they differ exactly in one bit.

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http://dx.doi.org/10.1016/j.ins.2014.05.052
0020-0255/

1.1. Motivating

Since edge faults or vertex faults may occur when a network is put into use, it is practically meaningful to consider faulty networks. An important property of hypercube is its fault tolerance. The problem of fault-tolerant embedding in the hypercubes has received much attention in recent years [2–4,7,19,21–23,25–28,31,32]. The cycle embedding problem, which deals with all the possible lengths of cycles in a given graph, has been studied in many interconnection networks, see [5–9,11–15,18,20–23,25,27–29,32,33]. Let f_e be the number of faulty edges in an n-dimensional hypercube Q_n. Li et al. [21] proved that each edge of Q_n lies on a fault-free cycle of every even length from 4 to 2^n, where $f_e \leqslant n-2$ and $n \geqslant 3$. Yang et al. [32] considered the Q_n with $f_e \leqslant 2n-5$ faulty edges under the condition that each vertex is incident to at least two fault-free edges and proved that Q_n has a fault-free cycle of every even length from 4 to 2^n. Liu and Wang [23] considered Q_n with $f_e \leqslant 3n-8$ faulty edges under the following two conditions: (1) each vertex is incident to at least two fault-free edges, and (2) every 4-cycle does not have any pair of non-adjacent vertices whose degrees are both two after removing the faulty edges, and proved that there still exists a fault-free Hamiltonian cycle in Q_n. We note that both Li et al. [21] and Yang et al. [32] proved that there exists a fault-free cycle of every even length from 4 to 2^n. However, Yang et al. [32] proved that the Q_n can tolerate more faulty edges under the condition that each vertex is incident to at least two fault-free edges. In this paper, we are motivated by the results of [21,32,23]. Our goal is to consider the Q_n under the conditions (1) and (2) and obtain a fault-free cycle of every even length from 4 to 2^n in Q_n when the number of faulty edges is at most $3n-8$.

1.2. Problem statement

Let F be the set of faulty edges in Q_n. Under the same conditions (1) and (2), in this paper we shall prove that Q_n with $|F| \leqslant 3n-8$ faulty edges has a fault-free cycle of every even length from 4 to 2^n, where $n \geqslant 5$.

1.3. Solution and analysis

For Q_5, we will prove the result is true by a single lemma. For Q_n $(n>5)$, firstly, we need to partition an n-dimensional hypercube Q_n into two $(n-1)$-cubes, always denoted by Q^0_{n-1} and Q^1_{n-1}. Throughout this paper, for an edge (u,v) in Q^i_{n-1}, we always use (u',v') to denote the corresponding edge in Q^{1-i}_{n-1}, where $\langle u,v,v',u',u\rangle$ is a 4-cycle and $i=0,1$. Note that both Q^0_{n-1} and Q^1_{n-1} are isomorphic to Q_{n-1}. See Fig. 1(a). Secondly, we use two ideas to prove the main result.

The first idea is shown in Fig. 1(b).

Step 1: By a known lemma, we can get a fault-free cycle C_1 of length $\ell(C_1)$ in Q^1_{n-1}, where $\ell(C_1)=2^{n-1}$.
Step 2: We find an edge, say (x,y), in C_1 such that $(x,x'),(y,y')$ and (x',y') are fault-free. (Note that (x,y) may be a faulty edge and $C_1-(x,y)$ is fault-free.) By a known lemma, we find a fault-free cycle C_0 of every even length $\ell(C_0)$ containing (x',y') in Q^0_{n-1}, where $4 \leqslant \ell(C_0) \leqslant 2^{n-1}$.
Step 3: Let $C'=[C_1-(x,y)]\oplus(x,x')\oplus(y,y')\oplus(x',y')$ with even length $\ell(C')=[\ell(C_1)-1]+3$. Then $\ell(C')=2^{n-1}+2$.

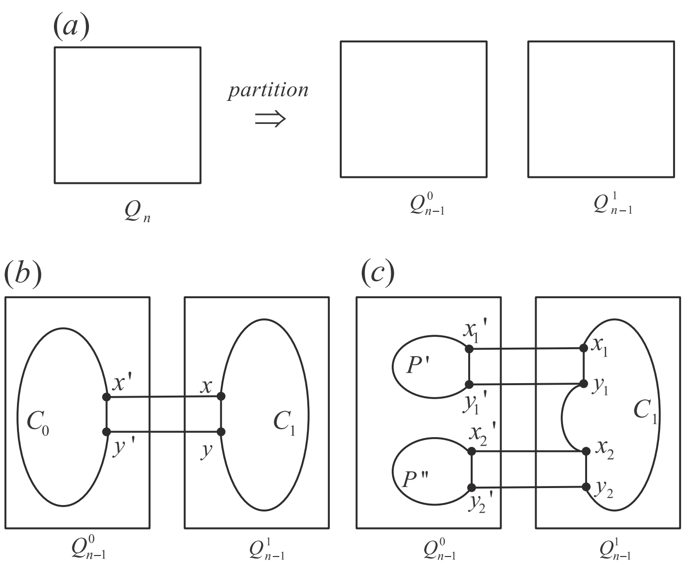

Fig. 1. The ideas of the main proof.