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Neural-network-based robust optimal control design for a class of uncertain nonlinear systems via adaptive dynamic programming ☆

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ABSTRACT

In this paper, the neural-network-based robust optimal control design for a class of uncertain nonlinear systems via adaptive dynamic programming approach is investigated. First, the robust controller of the original uncertain system is derived by adding a feedback gain to the optimal controller of the nominal system. It is also shown that this robust controller can achieve optimality under a specified cost function, which serves as the basic idea of the robust optimal control design. Then, a critic network is constructed to solve the Hamilton–Jacobi–Bellman equation corresponding to the nominal system, where an additional stabilizing term is introduced to verify the stability. The uniform ultimate boundedness of the closed-loop system is also proved by using the Lyapunov approach. Moreover, the obtained results are extended to solve decentralized optimal control problem of continuous-time nonlinear interconnected large-scale systems. Finally, two simulation examples are presented to illustrate the effectiveness of the established control scheme.

1. Introduction

In practical control systems, model uncertainties arise frequently and can severely degrade the closed-loop system performance. Hence, the problem of designing robust controller for nonlinear systems with uncertainties has drawn considerable attention in recent literature [43,15,31]. Lin et al. [15] showed that the robust control problem can be solved by studying the optimal control problem of the corresponding nominal system, but the detailed procedure was not presented. In [31], the authors developed an iterative algorithm for online design of robust control for a class of continuous-time nonlinear systems. However, the optimality of the robust controller with respect to a specified cost function was not discussed. In [43], the authors addressed the problem of designing robust tracking controls for a class of uncertain nonholonomic systems actuated by brushed direct current motors, while the research was not related with the optimality.

The starting point of the obtained strategy of this paper is optimal control. The nonlinear optimal control problem always requires to solve the Hamilton–Jacobi–Bellman (HJB) equation. Though dynamic programming has been a conventional method in solving optimization and optimal control problems, it often suffers from the curse of dimensionality, which was primarily due to the backward-in-time approach. To avoid the difficulty, based on function approximators, such as neural networks, adaptive/approximate dynamic programming (ADP) was proposed by Werbos [35] as a method to solve

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optimal control problems forward-in-time. Recently, the study on ADP and related fields have gained much attention from various scholars [1–10,12–14,16–25,28–30,32–34,36–38,40–42,44–46]. Lewis and Vrabie [13] stated that the ADP technique is closely related to the field of reinforcement learning. As is known to all, policy iteration is one of the basic algorithms of reinforcement learning. In addition, the initial admissible control is necessary when employing the policy iteration algorithm. However, in many situations, it is difficult to find the initial admissible control.

To the best of our knowledge, there are few results on robust optimal control of uncertain nonlinear systems based on ADP, not to mention the decentralized optimal control of large-scale systems. This is the motivation of our research. Actually, it is the first time that the robust optimal control scheme for a class of uncertain nonlinear systems via ADP technique and without using an initial admissible control is established. To begin with, the optimal controller of the nominal system is designed. It can be proved that the modification of optimal control law is in fact the robust controller of the original uncertain system, which also achieves optimality under the definition of a cost function. Then, a critic network is constructed for solving the HJB equation corresponding to the nominal system. In addition, inspired by the work of [5,24], an additional stabilizing term is introduced to verify the stability, which relaxes the need for an initial stabilizing control. The uniform ultimate boundedness (UUB) of the closed-loop system is also proved via the Lyapunov approach. Furthermore, the aforementioned results are extended to deal with the decentralized optimal control for a class of continuous-time nonlinear interconnected systems. At last, two simulation examples are given to show the effectiveness of the robust optimal control scheme.

2. Problem statement and preliminaries

In this paper, we study the continuous-time uncertain nonlinear systems given by

$$\dot{x}(t) = f(x(t)) + g(x(t))(\bar{u}(t) + \bar{d}(x(t))), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $\bar{u}(t) \in \mathbb{R}^m$ is the control vector, $f(\cdot)$ and $g(\cdot)$ are differentiable in their arguments with $f(0) = 0$, and $\bar{d}(x)$ is the unknown nonlinear perturbation. Let $x(0) = x_0$ be the initial state. We assume that $\bar{d}(0) = 0$, so that $x = 0$ is an equilibrium of system (1). As in many other literature, for the nominal system

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t), \tag{2}$$

we also assume that $f + gu$ is Lipschitz continuous on a set Ω in $\mathbb{R}^n$ containing the origin and that system (2) is controllable.

For system (1), in order to deal with the robust control problem, we should find a feedback control policy $\bar{u}(x)$, such that the closed-loop system is globally asymptotically stable for all uncertainties $\bar{d}(x)$. In this paper, we will show that this problem can be converted into designing an optimal controller for the corresponding nominal system with appropriate cost function.

Let $R \in \mathbb{R}^{m\times m}$ be a symmetric positive definite matrix. Then, we denote $d(x) = R^{1/2}\bar{d}(x)$, where $d(x) \in \mathbb{R}^m$ is bounded by a known function $d_M(x)$, i.e., $\|d(x)\| \leqslant d_M(x)$ with $d_M(0) = 0$. For system (2), in order to deal with the infinite horizon optimal control problem, we have to find the control policy $u(x)$, which minimizes the cost function given by

$$J(x_0) = \int_0^{\infty} \left\{ d_M^2(x(\tau)) + u^{\mathsf{T}}(x(\tau))Ru(x(\tau)) \right\} \mathrm{d}\tau. \tag{3}$$

Based on optimal control theory, the designed feedback control must not only stabilize the system on Ω, but also guarantee that the cost function (3) is finite. In other words, the control policy must be admissible [1,28]. Let $\Psi(\Omega)$ be the set of admissible controls on Ω. For any admissible control policy $u \in \Psi(\Omega)$, if the associated cost function (3) is continuously differentiable, then its infinitesimal version is the nonlinear Lyapunov equation given by

$$0 = d_M^2(x) + u^{\mathsf{T}}(x)Ru(x) + (\nabla J(x))^{\mathsf{T}}(f(x) + g(x)u(x)), \tag{4}$$

with $J(0) = 0$. In Eq. (4), the symbol $\nabla(\cdot) \triangleq \partial(\cdot)/\partial x$ is the notation of gradient operator, for example, $\nabla J(x) = \partial J(x)/\partial x$.

Define the Hamiltonian function of system (2) as follows:

$$H(x, u, \nabla J(x)) = d_M^2(x) + u^{\mathsf{T}}(x)Ru(x) + (\nabla J(x))^{\mathsf{T}}(f(x) + g(x)u(x)). \tag{5}$$

The optimal cost function of system (2) can be formulated as

$$J^*(x_0) = \min_{u\in\Psi(\Omega)} \int_0^{\infty} \left\{ d_M^2(x(\tau)) + u^{\mathsf{T}}(x(\tau))Ru(x(\tau)) \right\} \mathrm{d}\tau. \tag{6}$$

According to optimal control theory, the optimal cost function $J^*(x)$ satisfies the HJB equation

$$0 = \min_{u\in\Psi(\Omega)} H(x, u, \nabla J^*(x)). \tag{7}$$

Assume that the minimum on the right hand side of (7) exists and is unique. Then, the optimal control policy is

$$u^*(x) = -\frac{1}{2}R^{-1}g^{\mathsf{T}}(x)\nabla J^*(x). \tag{8}$$

Based on (5) and (8), the HJB Eq. (7) becomes