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Odd cycles embedding on folded hypercubes with conditional faulty edges

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ABSTRACT

Let FF_e be the set of $|FF_e| \leqslant 2n-5$ faulty edges in an n-dimensional folded hypercube FQ_n such that each vertex of FQ_n is incident to at least two fault-free edges, where $n \geqslant 4$ and n is even. Under this assumption, we show that every fault-free edge of FQ_n lies on a fault-free cycle of every odd length from $n+1$ to 2^n-1. In terms of the number of tolerant faulty edges and embedding odd cycles in FQ_n, our result improves not only the result in Xu and Ma (2006) where $|FF_e| = 0$, but also the previous best result gotten by Xu et al. (2006) where $|FF_e| \leqslant n-1$.

1. Introduction

The design of *interconnection networks* is essential to the parallel processing or distributed systems. There are various topologies of interconnection networks, see [10,26,27,31] as extensive references. Among them, the *hypercube* [31] has many excellent properties such as recursive structure, regularity, symmetry, small diameter, relatively short mean internode distance, low degree and link complexity, which are very important for designing massively parallel or distributed systems [26]. Many variants of the hypercube have been proposed, see [9,10,13] as examples. One of these variants is the *folded hypercube*, which is constructed from the hypercube by adding an edge of every pair of vertices with complementary addresses. It has been shown that, the folded hypercube is super to the regular hypercube in terms of many measurements [9,37].

An *embedding* of a *guest graph G* to another *host graph H* is a one-to-one mapping from the vertex set of G into the vertex set of H [26]. An embedding strategy provides us a scheme to emulate a guest graph on a host graph [16]. Linear arrays and rings, which are two of the most fundamental networks for parallel and distributed computation, are suitable for designing simple algorithms with low communication costs.

Since faults may happen in the real world networks, it is important to consider faulty networks. The fault-tolerant embedding has been proposed [1–8,11–16,19–25,27–30,32–43]. The fault-tolerant embedding on an n-dimensional folded hypercube FQ_n has been studied in many papers such as [3,5,11,14,15,17,18,23–25,29,37,40,41].

Let F_v and F_e be the sets of faulty vertices and faulty edges of Q_n, respectively. Tsai [34] showed that every fault-free edge of Q_n lies on a fault-free cycle of every even length from 4 to $2^n - 2|F_v|$ inclusive if $|F_e| + |F_v| \leqslant n-2$. Tsai [33] also proved

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that every fault-free edge in $Q_n - F_e$ lies on a cycle of every even length from 6 to 2^n under the condition that $|F_e| \leqslant 2n-5$ if each vertex is incident to at least two fault-free edges, where $n \geqslant 3$.

Let FF_e be the set of faulty vertices and faulty edges of FQ_n, respectively. Xu et al. [40] proved that every edge of FQ_n lies on a cycle of every even length from 4 to 2^n and a cycle of every odd length from $n+1$ to 2^n-1 when n is even. Xu et al. [41] considered FQ_n with $|FF_e| \leqslant n-1$ faulty edges and proved that each fault-free edge of FQ_n lies on a fault-free cycle of every even length from 4 to 2^n and a cycle of every odd length from $n+1$ to 2^n-1 when n is even. Under the condition that each vertex in FQ_n is incident to at least two fault-free edges, Kuo and Hsieh [23] proved that there exists a fault-free cycle of every even length from 4 to 2^n if $n \geqslant 2$ and every odd length from $n+1$ to 2^n-1 if $n \geqslant 2$ is even, where $|FF_e| \leqslant 2n-3$. Hsieh et al. [18] considered FQ_n with only one faulty vertex f and showed that $FQ_n - \{f\}$ contains a fault-free cycle of every even length from 4 to 2^n-2 if $n \geqslant 3$ and every odd length from $n+1$ to 2^n-1 if $n \geqslant 2$ is even. Cheng et al. [5] studied FQ_n with $|FF_v| \leqslant n-2$ faulty vertices and showed that if $n \geqslant 3$, then every edge of $FQ_n - FF_v$ lies on a fault-free cycle of every even length from 4 to $2^n - 2|FF_v|$; and if $n \geqslant 2$ is even, then every edge of $FQ_n - FF_v$ lies on a fault-free cycle of every odd length from $n+1$ to $2^n - 2|FF_v| - 1$. Hsieh [15] considered the FQ_n with both faulty edges and faulty vertices and proved that there exists a fault-free cycle of length at least $2^n - 2|FF_v|$ in FQ_n if $|F_v| + |F_e| \leqslant n-1$, where $n \geqslant 3$. Fu [11] improved the result in [15] and showed that $FQ_n - FF_e - FF_v$ contains a fault-free cycle of length at least $2^n - 2|FF_v|$ if $|FF_e| + |FF_v| \leqslant 2n-4$ and $|FF_e| \leqslant n-1$, where $n \geqslant 3$. Hsieh et al. [17] obtained the similar result as Fu [11] in $FQ_n - FF_e - FF_v$ with $|FF_v| + |FF_e| \leqslant 2n-4$ and $|FF_e| \geqslant n$ under the conditional edge faults.

In this paper, we consider the FQ_n with $|FF_e| \leqslant 2n-5$ faulty edges under the condition that each vertex is incident to at least two fault-free edges and prove that every fault-free edge lies on a fault-free cycle of every odd length from $n+1$ to 2^n-1, where $n \geqslant 4$ is even. As for embedding odd cycles in FQ_n, our result improves the results in [40,41] in terms of the number of fault-tolerant edges.

The remaining of this paper is organized as follows. In Section 2, necessary definitions and notations are introduced. In Section 3, we show our main results. Finally, concluding remarks are in Section 4.

2. Preliminaries

In this paper, a network topology is represented by a simple undirected graph, i.e., loopless and without multiple edges. A graph G is represented by $G = (V, E)$, where V is *vertex set* and E is *edge set*. An *edge e* is denoted by an unordered pair of vertices (u, v), where u and v are *end vertices* of e. u and v are said to be *incident to e*, and vice versa. Two edges are adjacent if they have a common vertex. Throughout this paper, *network* and *graph*, *node* and *vertex* are used interchangeably.

A *path P*, denoted by $\langle v_0, v_1, \ldots, v_n \rangle$, is a sequence of distinct vertices, where any two consecutive vertices are adjacent. v_0 and v_n are called *end vertices* of P. The length of a path, denoted by $l(P)$, is the number of edges in it. A cycle C is a path of length at least three with the same end-vertices.

An *isomorphism* from a graph G to a graph H is a bijection $\pi : V(G) \to V(H)$ such that $(u, v) \in E(G)$ if and only if $(\pi(u), \pi(v)) \in E(H)$. G is called isomorphism to H and denoted by $G \cong H$. An *automorphism* of G is an isomorphism from G to G. A graph is *edge transitive* (respectively, *node transitive*) if for any two edges e_1 and e_2 (respectively, v_1 and v_2), there exists an automorphism that maps e_1 to e_2 (respectively, v_1 to v_2).

An *n-dimensional hypercube* Q_n is an undirected graph with 2^n vertices, which are denoted by n-bit binary strings $u_n u_{n-1} \ldots u_1$, where $u_i \in \{0, 1\}$ for $1 \leqslant i \leqslant n$. Two vertices are adjacent if and only if they have only one different bit. The number of different bits between two vertices u and v are defined as *Hamming distance*, denoted by $d_H(u, v)$. The length of the shortest path between u and v in Q_n is denoted by $d_{Q_n}(u, v)$. Obviously, $d_H(u, v) = d_{Q_n}(u, v)$ in Q_n. An edge is along *dimension i* if its two end-vertices differ in the ith bit. Let $E_i = \{(u_n u_{n-1} \ldots u_{i+1} 0 u_{i-1} \ldots u_1, u_n u_{n-1} \ldots u_{i+1} 1 u_{i-1} \ldots u_1) | u_j \in \{0, 1\}$ for $1 \leqslant j \leqslant n$ and $j \neq i\}$. Q_n can be partitioned along dimension i ($1 \leqslant i \leqslant n$) into two $(n-1)$-cubes Q^0_{n-1} and Q^1_{n-1}, where Q^0_{n-1} (respectively, Q^1_{n-1}) is induced by the vertex set $\{u_n u_{n-1} \ldots u_{i+1} 0 u_{i-1} \ldots u_1 | u_j \in \{0, 1\}$ for $1 \leqslant j \leqslant n$ and $j \neq i\}$ (respectively, $\{u_n u_{n-1} \ldots u_{i+1} 1 u_{i-1} \ldots u_1 | u_j \in \{0, 1\}$ for $1 \leqslant j \leqslant n$ and $j \neq i$). For an edge e of dimension i, we also say Q_n can be divided along edge e of dimension i, which yields the same two $(n-1)$-cubes as those obtained after Q_n is divided along dimension i. For convenience, the vertex in Q^0_{n-1} (respectively, Q^1_{n-1}) is also denoted by $0x$ (respectively, $1x$), i.e., $0x = x_n x_{n-1} \ldots x_{i+1} 0 x_{i-1} \ldots x_1$ (respectively, $1x = x_n x_{n-1} \ldots x_{i+1} 1 x_{i-1} \ldots x_1$). Since Q_n is a bipartite graph, the vertices in the two bipartite sets are labeled by white vertices and black vertices, respectively.

An n-dimensional folded hypercube FQ_n can be constructed from an n-dimensional hypercube by adding an edge to every pair of vertices with complementary addresses, e.g., vertex $u = u_n u_{n-1} \ldots u_1$ and vertex $\bar{u} = \bar{u}_n \bar{u}_{n-1} \ldots \bar{u}_1$, where $u_n + \bar{u}_n = 1$. The set of all the complementary edges are denoted by E_a, i.e., $E_a = \{(u_n u_{n-1} \ldots u_1, \bar{u}_n \bar{u}_{n-1} \ldots \bar{u}_1) | u_j \in \{0, 1\}$ for $1 \leqslant j \leqslant n\}$. FQ_n is also denoted by $FQ_n = Q_n \cup E_a$. The 2-dimensional and 3-dimensional folded hypercubes are illustrated in Fig. 1.

The following lemmas are useful to our main proof.

Xu et al. [41] and Cheng et al. [5] respectively got the following three lemmas in FQ_n.

Lemma 1 [41]. *$FQ_n - E_i \cong Q_n$ for $i \in \{1, 2, \ldots, n, a\}$.*

Lemma 2 [41]. *Let FF_e be the set of faulty edges in FQ_n with $|FF_e| \leqslant n-1$. Then every fault-free edge lies on a fault-free cycle of every even length from 4 to 2^n and every odd length from $n+1$ to 2^n-1 if n is even.*