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Behavioral analysis of the leader particle during stagnation in a particle swarm optimization algorithm

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A B S T R A C T

Concept of the particle swarms emerged from a simulation of the collective behavior of social creatures. It gradually evolved into a powerful derivative-free optimization techniques, now known as Particle Swarm Optimization (PSO) for solving multi-dimensional, multi-modal, and non-convex optimization problems. The dynamics governing the movement of the particles in PSO has invoked a great deal of research interest over the last decade. Theoretical investigations of PSO has mostly focused on particle trajectories in the search space and the parameter-selection. This work looks into the PSO algorithm from the perspective of the leader particle and takes into account stagnation, a situation where particles are trapped at less coveted local optima, thus preventing them from reaching more coveted global optima. We show that the points sampled by the leader particle satisfy a simple mathematical relation which demonstrates that they lie on a specific line. We demonstrate the condition under which for certain values of the parameters, particles stick to exploring one side of the stagnation point only and ignore the other side, and also the case where both sides are explored. We also obtain information about the gradient of the objective function during stagnation in PSO. We provide a large number of machine simulations which support our claims over several ranges of the control parameters. This sheds light on possible modifications to the basic PSO algorithm which would help future researchers to work with even more efficient and state-of-the-art PSO variants.

1. Introduction

Kennedy and Eberhart [29,18] introduced the concept of function optimization by means of a particle swarm in 1995 [10,18]. In the basic PSO scheme, a "swarm" of particles move around in the search space influenced by continually better and improved positions discovered by other particles. PSO does not require any derivative information of the function to be optimized, uses only rudimentary mathematical operators, and is conceptually very simple.

Since its inception in 1995, PSO has attracted a great deal of attention of the researchers all over the globe resulting into nearly uncountable number of variants of the basic algorithm, theoretical and empirical investigations of the dynamics of the particles, parameter selection and control, and applications of the algorithm to a wide spectrum of real world problems from diverse fields of science and engineering [8,17,19,20,23,24,30–32,37].

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Being a stochastic search process, PSO is not free from false and/or premature convergence, especially over multi-modal fitness landscapes. Quite often, PSO does not work very well, and may require considerable tuning of it's parameters to specifically adapt to deceptive or intensely multi-modal optimization problems. For detailed analysis of parameter-tuning in PSO, see van den Bergh [1], Trelea [34], Shi and Eberhart [33], Carlisle and Dozier [7], and Clerc [9]. A few more important works in the field of PSO parameter-selection are [4,11,12,33].

Mathematical analysis of the dynamics of PSO has attracted a good deal of research interest over the last decade. Most of such analytical studies that have so far been undertaken focuses on the trajectories of the particles and the choices of parameters that will guarantee the convergence and stability of the trajectories. These issues have been addressed by [2,3,5,6,8,16,25,26,34,35]. The sampling distributions of the PSO were investigated in [9,21,27,28]. A stagnation-state analysis of the particle dynamics in PSO is an extremely important work which has not been comprehensively studied earlier. [21] reported an analysis of the dynamic equation of the leading (globally best) particle. However, the authors only provided a sufficient stable region for the parametric space of the PSO algorithm. The present work attempts to introduce some degree of rigor on the dynamics of the so-called "fittest" particle in the stagnation state of a PSO algorithm with deterministic control parameters. In doing so, we arrive at some interesting conclusions. The points sampled by the leader particle satisfy a mathematical relation which shows that they lie on a line. Moreover, the dynamics of the leader particle is wholly governed by a parameter-dependent function which may take three different forms. If we assume that the swarm stays in the stagnation state forever, then this function may be seen to converge to 0. Some simple mathematics leads us to the conclusion that this function, which we call $g(t)$, is never negative for certain choices of parameters. This case is significant in pinpointing the fact that if this happens, PSO loses it's exploratory nature and hence, that these parameters may not be good choices for PSO.

A dynamic, stagnation-state analysis of the leader particle also gives us some information about the relation between the line on which the points sampled by the leader particle lie and the gradient of the objective function. Our work leads us to the conclusion that the aforementioned line is either orthogonal to the direction of the gradient of the objective function or has a descent direction. As will be shown, these two cases are intricately linked to the choice of the parameters and also to the sign of the function $g(t)$.

The organization of the paper is as follows. Section 2 reviews the classical PSO algorithm. Section 3 is devoted to the dynamic analysis of the leader particle during stagnation. Section 4 presents the analysis according to which we conclude that the points sampled by the leader particle lie on a line. We also discuss the dynamic and limitative behaviors of the dominant particle here. Section 5 gives the relationship between the dynamic behavior discussed and the choice of parameters. We also see how restricting parameters to lie in certain sets (or their unions) imposes strict rules on the sign that $g(t)$ possesses. This is carried forward in Section 6 to obtain valuable information about the relationship between the line on which the points sampled by the leader particle lie and the gradient of the objective function. Section 7 provides some numerical verification of the theory that we have put forth. The work ends with a short discussion about prospective future developments which seamlessly follow from the conclusions garnered forthwith.

2. The PSO algorithm

This section provides a brief introduction to the basic PSO Algorithm. PSO maintains a swarm containing m particles where $m \in \mathbb{N}$ is a constant. Each particle is characterized by a position, a velocity and a knowledge of it's own neighborhood, utilizing which it can share information about the hitherto best position it has attained with the other particles traversing the experimental search space. This so-called "best" position is governed by the fitness value or simply fitness, which determines a particular particle's progress towards coveted local or global minima of the objective function under consideration. The particles traverse the search space dynamically and their movement is governed by the following fundamental equations:

$$v_{ij}(k+1) = \omega v_{ij}(k) + C_1(p_{ij}(k) - x_{ij}(k)) + C_2(g_j(k) - x_{ij}(k)) \tag{1}$$

and

$$x_{ij}(k+1) = x_{ij}(k) + v_{ij}(k+1) \tag{2}$$

where

(1) $\boldsymbol{x}_i(k) = (x_{i1}(k), x_{i2}(k), \ldots, x_{in}(k))^T$ is the position of the i-th particle at iteration k, and $\boldsymbol{v}_i(k) = (v_{i1}(k), v_{i2}(k), \ldots, v_{in}(k))^T, i = (1, 2, \ldots, m)$ is the velocity of the i-th at iteration k.
(2) $\boldsymbol{p}_i(k) = (p_{i1}(k), p_{i2}(k), \ldots, p_{in}(k))^T$ and $\boldsymbol{g}_i(k) = (g_{i1}(k), g_{i2}(k), \ldots, g_{in}(k))^T$ are the personal best position and the neighborhood best position of particle i at iteration k respectively. Their values are defined as follows:

$$\boldsymbol{p}_i(k) = \underset{0 \leqslant t \leqslant k}{\operatorname{argmin}} f(\boldsymbol{x}_i(t)) \tag{3}$$

and