ARTICLE IN PRESS

Information Sciences xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

A new method for deriving priority weights by extracting consistent numerical-valued matrices from interval-valued fuzzy judgement matrix

Feng Zhang [a,b,*], Joshua Ignatius [a,*], Chee Peng Lim [c], Yajun Zhao [d]

[a] School of Mathematical Sciences, Universiti Sains Malaysia, 11800 Minden, Penang, Malaysia
[b] College of Mathematics and Computer Science, Hebei University, Baoding 071002, China
[c] Centre for Intelligent Systems Research, Deakin University, Australia
[d] College of Physics Science and Technology, Hebei University, Baoding 071002, China

ARTICLE INFO

Article history:
Received 16 September 2012
Received in revised form 18 March 2014
Accepted 30 March 2014
Available online xxxx

Keywords:
Pairwise comparison matrix
Consistency
Priority weight vector
Fuzzy judgment matrix

ABSTRACT

It is important to derive priority weights from interval-valued fuzzy preferences when a pairwise comparative mechanism is used. By focusing on the significance of consistency in the pairwise comparison matrix, two numerical-valued consistent comparison matrices are extracted from an interval fuzzy judgement matrix. Both consistent matrices are derived by solving the linear or nonlinear programming models with the aid of assessments from Decision Makers (DMs). An interval priority weight vector from the extracted consistent matrices is generated. In order to retain more information hidden in the intervals, a new probability-based method for comparison of the interval priority weights is introduced. An algorithm for deriving the final priority interval weights for both consistent and inconsistent interval matrices is proposed. The algorithm is also generalized to handle the pairwise comparison matrix with fuzzy numbers. The comparative results from the five examples reveal that the proposed method, as compared with eight existing methods, exhibits a smaller degree of uncertainty pertaining to the priority weights, and is also more reliable based on the similarity measure.

1. Introduction

In the process of making decision analysis, the pairwise comparison methods for analyzing alternatives or criteria have been widely used [1–16,20–32,34–36,39–50]. Decision Makers (DMs) state their judgements in a sequence by comparing two alternatives or criteria at a time. The priority weights of the alternatives or criteria are generated from the paired comparison matrix. Owing to incomplete information resulting from complexity and uncertainty in the decision making environments, or lack of appropriate measurement units and scales, it is difficult for DMs to clearly translate their preferences in the form of numerical values with full confidence. Therefore, interval-valued judgements are more effective in capturing uncertainties that occurred during the paired comparison process. In the context of interval fuzzy preference

* Corresponding authors. Address: School of Mathematical Sciences, Universiti Sains Malaysia, 11800 Minden, Penang, Malaysia. Tel.: +60 164309138; fax: +60 4 657 0910 (F. Zhang). Tel.: +60 134400666; fax: +60 4 657 0910 (J. Ignatius).

E-mail addresses: amyfzhang@gmail.com (F. Zhang), joshua_ignatius@hotmail.com (J. Ignatius), chee.lim@deakin.edu.au (C.P. Lim), hbuzyj@126.com (Y. Zhao).

http://dx.doi.org/10.1016/j.ins.2014.03.120
0020-0255/

relations where each judgement is expressed by an additive-based interval, a number of approaches have been developed to derive the associated priority weights [8,14,21,40–43,50]. As an example, Xu and Chen [43] proposed linear programming models for both consistent and inconsistent interval fuzzy pairwise comparison matrices. The aim is to derive the interval priority weights by studying the relation between the crisp priority weights-based comparison matrix and the original interval fuzzy preferences. Motivated by the method proposed by Xu and Chen [43], Wang and Li [40] developed goal programming models for deriving interval weights from interval fuzzy preferences. They generated the weight-based interval matrix by using interval arithmetic on the interval priority weights. A similar idea was also presented by Jiang et al. [14] and Xia et al. [41]. However, there was a difference between these methods. Xu and Chen [43] used the crisp priority weights to construct the consistent matrix, while Jiang et al. [14], Wang and Li [40], and Xia et al. [41] employed the interval priority weights based on interval arithmetic. Later, Genç et al. [8] generated some simple formulae to derive the priority weights from an interval consistent fuzzy preference by extending the multiplicative transitivity of a fuzzy preference relation, which was defined by Tanino [34]. They argued that the proposed approach yields the same results from the method provided by Xu and Chen [43], but without having to solve the linear programming models. Liu et al. [21] defined a stricter definition of an interval consistent fuzzy preference, and proposed an algorithm to generate the interval priority weights from interval fuzzy preference relations by using the convex combination method.

We notice that the above mentioned mathematical programming models used to derive the priority weights are constructed by minimizing the difference between the weight-based consistent matrix and the original judgement matrix given by DMs. From the viewpoint of Saaty [30,31], the priority weights generated by extensively minimizing the differences between the ratio of the priority weights and the elements in the judgement matrix may distort the real solution to the problem under inconsistent circumstances. Recently, Guo and Wang [10] proposed different programming models to elicit dual interval weights by maximizing or minimizing the first ignorance of interval priority weights. This method avoids distorting the solution by minimizing the difference between the weight-based consistent matrix and the original judgement matrix given by DMs. The given interval judgements can be approximated by using the ratios of interval weights from the exterior and interior directions. However, such approximation may result in larger interval priority weights.

The key advantage of using interval-valued preference relations, instead of precise numerical values, is to tackle uncertainties inherited in DMs' judgements. If the numerical-valued consistent judgment matrix could be extracted from an interval pairwise comparison matrix, the priority weights derived from it could reflect DMs' preferences properly. Therefore, we attempt to directly discover the numerical-valued consistent matrices that are hidden in an interval pairwise comparison matrix. This constitutes the main rationale of this research.

In this paper, a new method is proposed to derive the priority weights by directly extracting the consistent numerical-valued additive-based pairwise comparison matrix. The method first elicits the numerical-valued consistent matrices from the consistent interval fuzzy pairwise comparison matrix by combining uncertainties in the interval preference relations and DMs' assessments. Subsequently, the priority weights are derived from the extracted numerical-valued consistent matrices. In the presence of inconsistency, the method first identifies alternatives or judgments that could possibly cause inconsistency, and then broadens the corresponding intervals to obtain the consistent matrices. One advantage of the proposed method is that it reduces the possibilities of producing distorted weights by minimizing the difference between the constructed consistent matrix and the original matrix. Another advantage is that the interval weights generated by the proposed method have a small "first ignorance" measure (an indicator defined in Definition 7, which is used in [9,10], to measure knowledge uncertainty pertaining to a set of weight intervals) as compared with seven other existing methods using five examples. Additionally, the proposed method can be viewed as a generalization of the method suggested by Liu et al. [21]. Besides that, the interval priority weights derived by our method are the form of normalized interval weights, which are usually neglected in most existing methods. In order to avoid confusion, the term *additive-based pairwise comparison*, instead of *fuzzy pairwise comparison*, is adopted throughout this paper.

This paper is organized as follows. Section 2 provides a review on basic concepts of multiplicative and additive consistent preference relations, and the associated theorems. To retain more information on the comparison between the interval priority weights, a probability-based method that considers the distribution of the interval is presented. In Section 3, a two-stage algorithm to generate the final priority weights from the interval additive-based comparison matrix is presented. In the first stage, programming models are constructed to elicit the numerical-valued consistent pairwise comparison matrices from the consistent or inconsistent interval-valued matrix. We demonstrate that the consistent matrix obtained by Liu et al. [21] is a special case of our method. Based on the obtained normalized interval priority weights, a row-column method is employed in the second stage to generate the final priority weights with different confidence degrees. The proofs for some necessary theorems are provided in Section 3. In Section 4, five numerical examples to illustrate the proposed method are presented, with the results compared and discussed. The outcome shows that the weights obtained by the proposed method are more accurate and valid, as compared with eight existing methods. Section 5 concludes the paper, along with some suggestions for further work.

2. Preliminaries

This subsection reviews the basic definitions and theorems with respect to the pairwise comparison matrix, and introduces a new definition for the consistent interval comparison matrix.