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On the periodic dynamics of memristor-based neural networks with time-varying delays

Jiejie Chen, Zhigang Zeng *, Ping Jiang

School of Automation, Huazhong University of Science and Technology, Wuhan 430074, China
Key Laboratory of Image Information Processing and Intelligent Control, Ministry of Education, Wuhan 430074, China

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A B S T R A C T

In this paper, we study the existence, uniqueness and stability of periodic solution for a wide class of memristor-based neural networks with time-varying delays. By employing the topological degree theory in set-valued analysis, differential inclusions theory and a new Lyapunov function method, we prove that the neural network has a unique periodic solution, which is globally exponentially stable. Moreover, we prove the existence, uniqueness and global exponential stability of equilibrium point for time-varying delayed memristor-based neural networks with constant coefficients. The obtained results improve and extend previous works on memristor-based or usual neural network dynamical systems with continuous or discontinuous right-hand side. Finally, two numerical examples are provided to show the applicability and effectiveness of our main results.

1. Introduction

As is well known, the periodic oscillation in the neural networks is an interesting dynamical behavior, as many biological and cognitive activities (e.g., heartbeat, respiration, mastication, locomotion, and memorization) require repetition. It has been found applications in associative memories [22], pattern recognition [35,8], learning theory [23,27], and robot motion control [15]. In the past few decades, the studies of the periodic oscillation of various neural networks such as the Hopfield network, cellular neural networks, and bidirectional associative memories are all reported in the literature. An equilibrium point can viewed as a special case of periodic solution with an arbitrary period or zero amplitude. In this sense, the analysis of periodic oscillation of neural networks is more general than the stability analysis of equilibrium points. It also knows that time delay is the main cause of in stability and poor performance of networks. Therefore, it is very important to study the global stability and the global periodicity of neural networks with time-varying delays. There have been many results on the stability analysis of recurrent neural networks with and without delays (see, for instance, [43,19,6,7,16,26] and references therein). In addition, the stability and robust stability problem for stochastic neural networks with delays have been addressed in [38,39]. Other applications about neural networks can see also [29,18].

In 1971 Prof. Chua [5] postulated theoretically the existence of a fundamental two-terminal passive device, called memristor (as a contraction of memory and resistor) using symmetry logical reasonings. For the device a nonlinear relationship links charge and flux. The resistance of a current-controlled (voltage-controlled) memristor is uniquely determined by the time history of current through it (voltage across it) and is indefinitely storable by the device once the controlling source

* Corresponding author at: School of Automation, Huazhong University of Science and Technology, Wuhan 430074, China.
E-mail addresses: chenjiejie118@gmail.com (J. Chen), hustzgzeng@gmail.com (Z. Zeng).

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is turned off. In 2008 credit for the first conscious experimental observation of memristor behavior in nature was given to researchers at HP Labs, that showed how the properties of a memristor (theoretically devised by Chua) are embodied in a metal-oxide film-metal nanoscale device [25]. From then on the memristor behavior is more and more noticeable as new technology process nodes are introduced in integrated circuit design [36,37], where the memristor may be used as a non-volatile memory switch [17,2,24]. We know that a Hopfield neural network model can be implemented in a circuit where the connection weights are implemented by resistors, motivated by these facts, recently, by using memristors instead of resistors, many scholar have studied a new model, where the connection weights change according to its state, i.e., a state-dependent switching recurrent neural networks, which is said to be the memristor-based recurrent neural networks (see, for instance, [30,14,32,31,33,34,40–42]). However, to our knowledge, there are very few works dealing with the periodicity of the memristor-based recurrent neural networks with time-varying delays (see [42]). And the global periodicity and global stability of memristor-based recurrent neural networks with time-varying delays plays important roles in many potential applications, e.g., super-dense nonvolatile computer memory and neural synapses.

Motivated by the above discussions, based on the works in [30,14,32,31,33,34,40–42], our objective in this paper is to study the existence and exponential stability of the periodic solutions for a memristor-based neural networks with time-varying delays. By employing the topological degree theory in set-valued analysis, differential inclusions theory and a new Lyapunov function method, we study the existence, uniqueness and exponential stability of the periodic solution for the networks.

The rest of the paper is organized as follows. A delayed memristor-based neural networks is introduced and some necessary definitions are given in Section 2. The existence of periodic solutions of the system are investigated in Section 3. Our approaches are based on the differential inclusions theory and topological degree theory in set-valued analysis. In Section 4. we shall study the uniqueness and global exponential stability of the ω-periodic solution for the system. Especially, when the system is autonomous, we will give the sufficient conditions, uniqueness and global exponential stability of equilibrium point of the autonomous system. In Section 5. two examples and simulations are obtained to show the effectiveness of the theoretical results given in the previous sections. Finally, the paper is concluded in Section 6.

2. Model description and preliminaries

In this paper, referring to some relevant works in [30,14,32,31,33,34,40–42], which deal with the detailed construction of some general classes of memristor-based recurrent neural networks from the aspects of circuit analysis and memristor physical properties, we consider a class of memristor-based recurrent neural networks described by the following equation:

$$\frac{dx_i(t)}{dt} = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t, x_j(t))f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t, x_j(t-\tau_{ij}(t)))g_j(x_j(t-\tau_{ij}(t))) + I_i(t), \tag{2.1}$$

for $i = 1, 2, \ldots, n$, where n corresponds to the number of units in a neural network; $x_i(t)$ denotes the state variable associated with the ith neuron; $f_{ij}(x_i(t))$ and $g_{ij}(x_j(t-\tau_{ij}(t)))$ denotes the output of the jth unit at time t and $t-\tau_{ij}(t)$, respectively; $c_i(t) > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs at time $t; I_i(t)$ denotes the external bias on the ith unit at time $t; \tau_{ij}(t)$ corresponds to the transmission delay of the ith unit along the axon of the jth unit at time t and is continuously differentiable function satisfying

$$0 \leqslant \tau_{ij}(t) \leqslant \tau, \dot{\tau}_{ij}(t) \leqslant \tau_{ij}^D < 1, \tag{2.2}$$

in which $\tau = \max_{1 \leqslant i,j \leqslant n}\{\max_{t\in[0,\omega]}\tau_{ij}(t)\}, \tau$ and τ_{ij}^D are all nonnegative constants; $a_{ij}(t, x_j(t))$ and $b_{ij}(t, x_j(t-\tau_{ij}(t)))$ are memristor-based weights, denote the strengths of the jth unit on the ith unit at time t and time $t-\tau_{ij}(t)$, respectively, which are defined as following

$$a_{ij}(t,\xi) = \begin{cases} \hat{a}_{ij}(t), & |\xi| > T_j, \\ \check{a}_{ij}(t), & |\xi| < T_j, \end{cases} \tag{2.3}$$

and

$$b_{ij}(t,\xi) = \begin{cases} \hat{b}_{ij}(t), & |\xi| > T_j, \\ \check{b}_{ij}(t), & |\xi| < T_j, \end{cases} \tag{2.4}$$

for $i,j = 1, 2, \ldots, n$ and $t \in R$, where $a_{ij}(t, \pm T_j) = \hat{a}_{ij}(t)$ or $\check{a}_{ij}(t)$ and $b_{ij}(t, \pm T_j) = \hat{b}_{ij}(t)$ or $\equiv \check{b}_{ij}(t)$ for $i,j = 1, 2, \ldots, n$, switching jumps $T_j > 0$ are all constants, $\hat{a}_{ij}(t), \check{a}_{ij}(t), \hat{b}_{ij}(t)$ and $\check{b}_{ij}(t)$ are all continuous functions.

In system (2.1), if $a_{ij}(t,\xi) = a_{ij}(\xi), b_{ij}(t,\xi) = b_{ij}(\xi), c_i(t) \equiv c_i$, $\hat{a}_{ij}(t) \equiv \hat{a}_{ij}, \check{a}_{ij}(t) \equiv \check{a}_{ij}, \hat{b}_{ij}(t) \equiv \hat{b}_{ij}$, $\check{b}_{ij}(t) \equiv \check{b}_{ij}$ and $I_i(t) \equiv I_i$ for all $i,j = 1, 2, \ldots, n$, where $c_i, \hat{a}_{ij}, \check{a}_{ij}, \hat{b}_{ij}, \check{b}_{ij}$ and I_i are constants, then we obtain the following autonomous memristor-based neural networks with time-varying delay corresponding to system (2.1):

$$\frac{dx_i(t)}{dt} = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij}(x_j(t))f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_j(t-\tau_{ij}(t)))g_j(x_j(t-\tau_{ij}(t))) + I_i, \tag{2.5}$$