Information Sciences xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Numerical solution of systems of second-order boundary value problems using continuous genetic algorithm

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A R T I C L E I N F O

Article history:
Received 16 December 2011
Received in revised form 20 August 2013
Accepted 29 March 2014
Available online xxxx

Keywords:
Continuous genetic algorithm
System of boundary value problem
Finite difference approximation

A B S T R A C T

In this paper, continuous genetic algorithm is introduced as an efficient solver for systems of second-order boundary value problems where smooth solution curves are used throughout the evolution of the algorithm to obtain the required nodal values of the unknown variables. The solution methodology is based on representing each derivative in the system of differential equations by its finite difference approximation. After that, the overall residue for all nodes in the given system of differential equations is formulated. The solution to the system of differential equations is then converted into the problem of minimizing the overall residue or maximizing the fitness function based on the nodal values generated from the genetic operators. Three numerical test problems including linear and nonlinear systems were analyzed to illustrate the procedure and confirm the performance of the proposed method. In addition to that, a convergence and sensitivity analysis to genetic operators and control parameters of the algorithm has been carried out. The numerical results show that the proposed algorithm is a robust and accurate procedure for solving systems of second-order boundary value problems. Furthermore, the obtained accuracy for the solutions using CGA is much better than the results obtained using some modern methods.

1. Introduction

Systems of second-order boundary value problems (BVPs) which are a combination of systems of second-order ordinary differential equations subject to given boundary conditions occur frequently in applied mathematics, theoretical physics, engineering, biology, mathematical modeling of real world problems in which uncertainty or vagueness pervades, and so on [19,22,33,38,40,44,59,60]. In fact, accurate and fast numerical solutions for systems of second-order BVPs are of great importance due to its wide application in scientific and engineering research.

The objective of numerical methods is to solve complex numerical problems using only the simple operations of arithmetic, to develop and evaluate methods for computing numerical results from given data. The methods of computation are called algorithms. An algorithm is a finite sequence of rules for performing computations on a computer such that at each instant the rules determine exactly what the computer has to do next. Numerical methods tend to emphasize the implementation of the

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http://dx.doi.org/10.1016/j.ins.2014.03.128
0020-0255/© 2014 Published by Elsevier Inc.

algorithms. Thus, numerical methods are methods for solving problems on computers by numerical calculations, often giving a table of numbers and/or graphical representations or figures.

Numerical methods are becoming more and more important in mathematical and engineering applications not only because of the difficulties encountered in finding exact analytical solutions, but also because of the ease with which numerical techniques can be used in conjunction with modern high-speed digital computers. There exist a large number of systems of second-order BVPs in science and engineering, whose solutions cannot easily be obtained by the well-known analytical methods. For such systems, we can obtain approximate solutions for using numerical methods under the given boundary conditions.

It is worth stating that in many cases, since systems of second-order BVPs are often derived from problems in physical world, existence and uniqueness are often obvious for physical reasons. Notwithstanding this, a mathematical statement about existence and uniqueness is worthwhile. Uniqueness would be of importance if, for instance, we wished to approximate the solutions. If two solutions passed through a point, then approximations could very well jump from one solution to the other with misleading consequences. Therefore, we assume that the systems of second-order BVPs to be solved numerically have unique solutions on the given interval.

Investigation about systems of second-order BVPs numerically is scarce. Recently, many authors have discussed the numerical solvability for the coupled system of second-order BVPs using some of the well-known methods. It is worth noting that the coupled system of second-order BVPs is just a special case of the system that we propose in this work. However, the reader is asked to refer to [18,21,26,27,31,32,43,52] in order to know more details about these methods, including their types, history, motivation for use, characteristics, and applications.

In this paper, we introduce a novel method based on the use of CGA for numerically approximating a solution of systems of second-order BVPs in which the given boundary conditions can be involved. The term "continuous" is used to emphasize that the continuous nature of the optimization problem and the continuity of the resulting solution curves. The new method has the following characteristics:

1. It does not require any modification while switching from the linear to the nonlinear case; as a result, it is of versatile nature.
2. This approach does not resort to more advanced mathematical tools; that is, the algorithm is simple to understand, implement, and should be thus easily accepted in the mathematical and engineering application's fields.
3. The algorithm is of global nature in terms of the solutions obtained as well as its ability to solve other mathematical and engineering problems.
4. The present technique is accurate, need less effort to achieve the results, and is developed especially for nonlinear case.

The organization of this paper is as follows: in the next section, a short introduction to optimization techniques is presented. In Section 3, we formulate the system of second-order BVPs as an optimization problem based on the fitness function. Section 4 covers the description of CGA in details. Numerical results are given in Section 5 in order to verify the mathematical simulation of the proposed algorithm. Convergence and statistical analysis are provided in Section 6 in order to capture the behavior of the solutions and to discover the effect of system parameters on the convergence speed of the proposed algorithm. Finally, concluding remarks are presented in Section 7.

2. Optimization: introduction and basic concepts

We are confronted not only with optimization in mathematics and engineering, but also in almost every aspect of life. Maybe, optimization is most important process of life. When we learn walking, talking, or anything else, our brain is doing some optimization. When we observe natural evolution, we have the impression that some optimization is done. When we look at the development of theories, we can consider this as an optimization process. These observations are fascinating and also inspiring for the development of numerical optimization procedures, especially when difficult, highly complex problems have to be tackled.

In mathematics, the term optimization refers to the study of problems in which one seeks to minimize or maximize a real function that satisfies the prevailing constraints by systematically choosing the values of real or integer variables from within an allowed set. Engineers and scientists present a new idea and optimization improves that idea. It involves trying variations on an initial concept and using the information gained to improve the idea. Over years, optimization techniques have been applied in diverse fields ranging from operations and economics to engineering and medicine [20,23,24,28,30,46,50,58].

Optimization techniques have undergone a great deal of change in recent times which allows us to apply these techniques to the most complex problems of today's world. The following steps summarize the general procedure used to formulate and solve optimization problems:

1. Analyze the process itself to identify the process variables and specific characteristics of interest, i.e., make a list of all the variables.
2. Determine the criterion for optimization and specify the objective function in terms of the above variables together with coefficients.