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Non-asymptotic fuzzy estimators based on confidence intervals

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ABSTRACT

In this paper, we study and generalize the method of fuzzy estimation which is based on confidence intervals. We provide definitions and the corresponding formulas. The method we propose allows new additional ways in constructing fuzzy estimators. In particular, instead of constructing fuzzy estimators as the exact set of confidence intervals of the parameters to be estimated, we generalize them, in such a way that we eliminate discontinuities, and ensure compact support preserving the statistically derived triangular shape of the estimators. Our approach is particularly useful in critical situations, where subtle fuzzy comparisons between almost equal statistical quantities have to be made. An example of this application is provided.

1. Introduction

Statistical estimation is one of the main techniques used in inferential statistics. A statistical model may be perceived as a particular type of description of a probability distribution. Such models contain parameters whose values are estimated from samples, a required task to proceed in statistical data analysis [1].

With the growth of fuzzy methodologies, the integration of statistics with fuzzy set theory has attracted considerable attention [6,12,9,22,19,20,3]. Kruse [12] proposed the use of fuzzy linguistic values for estimating the parameters of statistical distributions in a fuzzy way. Gil et al. [9] introduced the notion of fuzzy Bayes' estimator for real valued parameters. Yao and Hwang [22] investigated point estimators of fuzzy random variables for vague data. Wagner [19] described how to construct a fuzzy number on the basis of confidence intervals from a given sample but only for a determined set of α-cuts. Wu [20] proposed the concept of fuzzy estimators constructed from a family of closed intervals under the consideration of fuzzy random variables.

Using a different approach, Buckley [3] proposed a fuzzy estimation method in order to construct fuzzy numbers for parameters in probability density (mass) functions that have been estimated from random samples, using all confidence intervals. Various applications of this approach to fuzzy estimation can be found in the literature (see [16,10,17,11]).

Falsafain et al. [7] studied the construction of explicit and unique membership functions for such fuzzy estimators and recently Falsafain and Taheri [8] presented an improvement for non-symetric distributions. In this paper we generalize the fuzzy estimation approach established by Buckley [3].

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Our motivation in based in the need to compare two almost equal statistical quantities. In such critical situations, the shape of the fuzzy numbers that are being produced is very important. While in the literature [3,7,8], fuzzy estimation being an extension of confidence interval estimation makes an assumption that the shape of fuzzy estimators is bound to the exact set of confidence intervals, we show that this is not the only possible way to construct fuzzy estimators. We provide a generalization which not only includes previous approaches as a subcase, but also permits the construction of many families of such fuzzy estimators which we call them non-asymptotic fuzzy estimators (NAFEs). The new generalized definition guarantees that NAFE are proper fuzzy numbers, preserving the statistically derived triangular shape and are appropriate for subtle fuzzy comparisons.

An extra benefit of our proposed method is that one can now choose how close to be to the original point estimation, that is how strict is the fuzzy estimation. We show that the comparison of fuzzy estimators is greatly affected by their shapes, so a criterion is required to decide which shape is appropriate and we introduce the false alarm rate index. As an experiment, we tested the behavior and effectiveness of seven families of NAFE. The results have shown that by implementing NAFE on almost equal statistical quantities, we can significantly reduce the false alarm rate, i.e. we can correctly find which option is slightly better than the other when using either point or confidence interval estimation or fuzzy estimation. Thus our own non-asymptotic fuzzy estimators clearly address the issue of comparing almost equal quantities. An application of NAFE in the sensitive area of financial decisions is presented. For more applications of NAFE see [4,18].

The remainder of this paper is organized as follows: In Section 2.1 we introduce notions and definitions from Fuzzy Set theory. In Section 2.2 we review the literature and particularly Buckley's method of fuzzy estimation. In Section 3 we present the new method of non-asymptotic fuzzy estimators, in Section 3.2 we develop the corresponding membership function of non-asymptotic fuzzy estimators and in Section 3.3 we propose membership functions for some common statistical models. In the next Sections 4, 5 we discuss a fuzzy comparison method and provide the falst alarm criterion for the selection of the appropriate shape of the estimators. Section 6 is an application. Our conclusions follow in Section 7.

2. Fuzzy estimation based on confidence intervals

2.1. Fuzzy sets notions and definitions

Before presenting the fuzzy estimation method, we introduce the relevant mathematical notations and the basic notions from the fuzzy set theory.

Let X be a universal set. Every function of the form $\tilde{A}: X \to [0,1]$ is called a *fuzzy set* or a *fuzzy subset* of X, where $\tilde{A}(x)$ is interpreted as the *membership degree* of x in the fuzzy set $\tilde{A}$. We place a tilde over a fuzzy set symbol so as to distinguish it from classical set. Classical sets are also called *crisp* sets and are special cases of fuzzy sets where $\tilde{A}(x)$ is only zero or one.

The α-cuts of a fuzzy set $\tilde{A}$ are defined by the sets

$$^{\alpha}\tilde{A} = \left\{x \in \Re : \tilde{A}(x) \geqslant \alpha\right\}, \alpha \in (0,1] \tag{1}$$

while its support $^{0}\tilde{A}$ is the closure in the topology of X of the union of all the α-cuts [5], that is

$$^{0}\tilde{A} = \overline{\cup_{\alpha\in(0,1]}{}^{\alpha}\tilde{A}} = \overline{\left\{x : \tilde{A}(x) > 0\right\}}. \tag{2}$$

It is known that α-cuts uniquely determine the fuzzy set $\tilde{A}$. We say that $\tilde{A}$ is a fuzzy number if the following conditions hold:

1. $\tilde{A}$ is normal, that is there exists $x \in \Re$ such that $\tilde{A}(x) = 1$,
2. $\tilde{A}$ is a convex fuzzy set, that is for every $t \in [0,1]$ and $x_1, x_2 \in \Re$, we have
$$\tilde{A}((1-t)x_1 + tx_2) \geqslant \min\left\{\tilde{A}(x_1), \tilde{A}(x_2)\right\},$$
3. $\tilde{A}$ is upper semi-continuous on $\Re$, i.e. $\forall x_0 \in \Re$ and $\forall \varepsilon > 0$ there exists a neighborhood $V(x_0)$ such that $u(x) \leqslant u(x_0) + \varepsilon, \forall x \in V(x_0)$,
4. the support of $\tilde{A}$, Eq. (2), is compact.

We also mention that the α-cuts of a fuzzy number $\tilde{A}$ can be written as intervals of the form

$$^{\alpha}\tilde{A} = [A_1(\alpha), A_2(\alpha)] \tag{3}$$

where $A_1(\alpha), A_2(\alpha)$ can be regarded as functions on $[0,1]$ [15]. Then,

1. $A_1(\alpha)$ is non-decreasing and left continuous,
2. $A_2(\alpha)$ is non-increasing and left continuous,
3. $A_1(\alpha) \leqslant A_2(\alpha)$.

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