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A triangular type-2 multi-objective linear programming model and a solution strategy

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ABSTRACT

We consider a multi-objective linear programming model with type-2 fuzzy objectives. The considered model has the flexibility for the user to specify the more general membership functions for objectives to reflect the inherent fuzziness, while being simple and practical. We develop two solution strategies with reasonable computing costs. The additional cost, as compared to the type-1 fuzzy model, is indeed insignificant. These two algorithms compute Pareto optimal solutions of the type-2 problems, one being based on a maxmin approach and the other on aggregating the objectives. Finally, applying the proposed algorithms, we work out two illustrative examples.

1. Introduction

In crisp operation research, it is assumed that we know the objective function $f(x)$, whose values present what is best for the decision maker(s). Often, uncertainties have also been introduced in mathematical programming problems; in these cases, we cannot predict the exact outcome of objective function of a decision x, and outcome depends on the unknown factors. If experts can provide narrow intervals that contain x with certain degree of confidence α, then we can use type-1 fuzzy objective functions [37]. For definitions and solution methods for such problems, see [28]. However, sometime because of degree of uncertain information, experts cannot present fuzzy objective with deterministic membership grades [58]. Moreover, type-2 fuzzy sets offer a higher dimension to the problem so that more means would be available to accommodate for the inherent uncertainties than the type-1 fuzzy sets, having crisp membership functions (see [32,39]). In fact, when determination of exact membership functions may not be possible, type-2 fuzzy sets may turn to effective [48]. For example, Hisdal [20] believes that "increasing fuzziness in a description means increased ability to handle inexact information in logically correct manner", or John [23] asserts that "type-2 fuzzy sets allow for linguistic grades of membership, thus assigning in knowledge representation, and they also offer improvement of inference in constraint of type-1 sets".

Different approaches are developed for tackle more uncertainty than what can be modeled by type-1 fuzzy in mathematical optimization models. For example, various studies used interval type-2 fuzzy to model parameters of mathematical optimization models [3,22].

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In this study, we present a multi-objective model with crisp parameters and type-2 membership functions for the objectives and then discuss the solving approaches. As discussed before type-2 fuzzy can be used when experts are not certain about type-1 fuzzy membership of objective functions. To the best knowledge of the authors, this study is the first work that considers type-2 memberships functions for objectives.

A drawback for the use of type-2 fuzzy sets in multi-objective models may be due to the extra computations needed for computing solutions (see [46]), but this should not be a deterrence as the extra capability offered by these models turns to be compensatory. In fact, it is a usual practice to accept the complications caused by considering fuzzy weights for multi-objective programming problems in order to attain more practical models. Moreover, the proposed approaches have reasonable computing costs and the additional cost, as compared to the type-1 fuzzy model, is indeed insignificant.

Next, we present some preliminaries and necessary notations and definitions about multi-objective linear models and type-2 fuzzy, with a brief review of type-2 fuzzy linear programming approaches in Section 2. In Section 3, we propose the type-2 fuzzy multi-objective linear models and solution methods. We work out two illustrative examples in Section 4 and conclude in Section 5.

2. Preliminaries

2.1. Multi-objective programs using type-1 fuzzy sets

Consider the multi-objective problem,

$$\begin{aligned} &Min\ Z(x) = (z_1(x), z_2(x), \ldots, z_k(x)) \\ &s.t.\quad x \in X = \{x | Ax \leqslant b,\ x \geqslant 0\}, \end{aligned} \tag{1}$$

where $z_i = (z_{i1}, z_{i2}, \ldots, z_{in})$, $i = 1,2,\ldots,k$, $x = (x_1, \ldots, x_n)^T$, $b = (b_1, \ldots, b_m)^T$, and $A = [a_{ij}]$ is an $m \times n$ matrix.

There are various interpretations and corresponding solution methods for (1). One approach assumes prespecified priorities for the objective functions [35]. Methods based on this approach are called "scalarization methods". Other approaches in this class include "weighting" [52], "ε-constraint" [18] and "maxmin" [2] methods. We have also proposed an effective maxmin method; see Lin et al. [34].

Other methods are called "non-scalarization methods". In this class of methods, goal programming approach is prominent [19,44,49]; for a survey of various such multiobjective models and methods see [56].

Here, we begin with an approach due to Lai and Hwang [29]. To construct the membership functions, first compute z^* and z^- as the min and max values of the z_i as follows:

$$Z^* = \{z_1^*, \ldots, z_k^*\} = [\min z_1, \ldots, \min z_k] \tag{2}$$

$$Z^- = \{z_1^-, \ldots, z_k^-\} = [\max z_1, \ldots, \max z_k] \tag{3}$$

The resulting two vectors are named to be the ideal and the negative ideal solutions, respectively. Associated with (2) and (3), an initial solution for the objective vector can be determined by the decision maker, say $o = [o_1, \ldots, o_k]$, and then the membership functions for the objectives functions can be defined as:

$$u_i(z_i(x)) = \begin{cases} 1, & z_i(x) < z_i^*(x) \\ 1 - \frac{z_i^*(x) - z_i(x)}{z_i^*(x) - o_i}, & z_i^*(x) \leqslant z_i(x) \leqslant o_i, \quad i = 1, \ldots, k. \\ 0, & z_i(x) \geqslant o_i. \end{cases} \tag{4}$$

It is clear that the initial objective vector should be decided somewhere between the two vectors z^* and z^-. Of course, one can use the negative ideal solution vector as the initial objective vector. With this selection, we have:

$$u_i(z_i(x)) = \begin{cases} 1, & z_i(x) < z_i^*(x) \\ 1 - \frac{z_i^*(x) - z_i(x)}{z_i^*(x) - z_i^-(x)}, & z_i^*(x) \leqslant z_i(x) \leqslant z_i^-(x),\ i = 1, \ldots, k. \\ 0, & z_i(x) \geqslant z_i^{-*}(x). \end{cases} \tag{5}$$

Using the membership function u_i and following the symmetry principle of Bellman and Zadeh[1], the following problem may thus be posed:

$$\begin{aligned} &Max\ \underset{i=1,\ldots,k}{\text{Min}} \{u_i(x)\} \\ &s.t.\quad x \in X. \end{aligned} \tag{6}$$

Problem (6) is equivalent to

$$\begin{aligned} &Max\ \lambda \\ &s.t.\ \lambda \leqslant u_i(x),\ i = 1, \ldots, k, \\ &\qquad x \in X. \end{aligned} \tag{7}$$