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A note on modal logic and possibility theory ☆

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ABSTRACT

There are two theories in which the concept of possibility plays an important role—modal logic and possibility theory. The roles are different, and so are the agendas of modal logic and possibility theory. To gain an insight into the differences, a very simple model of modal logic is constructed. The model has the structure of a finite-state system, referred to as the FS-model. The FS-model may be viewed as a simple interpretation of Kripke model—an interpretation which is easy to understand. The FS-model is in the spirit of graph models of modal logic. The FS-model readily lends itself to generalization. Concrete versions of the FS-model serve as examples.

1. Introduction

There are two theories in which the concept of possibility plays an important role—modal logic and possibility theory. The role of the concept of possibility in modal logic is very different from its role in possibility theory. Interestingly, on a deeper level, a striking similarity comes to light. In large measure, what follows is motivated by the question: In what basic ways does the concept of possibility in modal logic differ from the concept of possibility in possibility theory?

Modal logic is a deep theory which is not easy to understand. [2] For comparison of modal logic with possibility theory, what is constructed in this note is a very simple abstract model which has the structure of a finite-state system, referred to as the FS-model. The best known model of modal logic is Kripke model. There are many models which are equivalent to Kripke model. [1,7] The FS-model may be viewed as a simple interpretation of Kripke model and is in the spirit of graph models of modal logic. The FS-model is easy to understand and readily lends itself to generalization. A summary of the FS-model is described in the following. It should be underscored that this note touches upon only elementary aspects of modal logic and possibility theory.

2. FS-model

The FS-model has five principal components.

(1). A collection of states, $W = (w_1, \ldots, w_n)$. W is referred to as the state space of FS. In the abstract model, the states are simply symbols with no meaning. The states do have meaning in concrete versions of the FS-model.

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(2). A collection of collections of inputs, with each state, w_i, associated with a collection of inputs, $U_i = (u_{i1}, \ldots, u_{ik})$, with k dependent on w_i, $k = k(i)$. (Fig. 1)
(3). State-transition function, f,

$$s_{t+1} = f(s_t, u_t),$$

where s_t is the state at time t, s_{t+1} is the next state, and u_t is the input at time t. s_t and s_{t+1} take values in the state space, W. The state-transition function is represented as a state diagram (Fig. 2).

(4). A proposition, p. A truth function, $\mathrm{tr}(p, w_i)$, associates with each state, w_i, the truth value, t_i, of p in w_i, $t_i = \mathrm{Tr}(p, w_i)$. If p is a crisp proposition, t_i is either true (1) or false (0). If p is a fuzzy proposition, t_i takes values in the unit interval.
(5). A target set, $T(p)$, is a collection of what are called target states. A target state, w_j, is a state in which $t_j = 1$. Thus, $T(p)$ is the collection of all states in which p is true. The target set is defined by p, that is, p serves to define the target set, $T(p)$. A state, w_j, satisfies $T(p)$, if w_j is a target state. Thus, $T(p) = \{w_j | \mathrm{tr}(p, w_j) = 1)\}$. The target set for not p is the complement in W of the target set for p (Fig. 3).

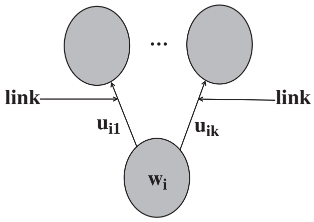

Fig. 1. Inputs in state w_i.

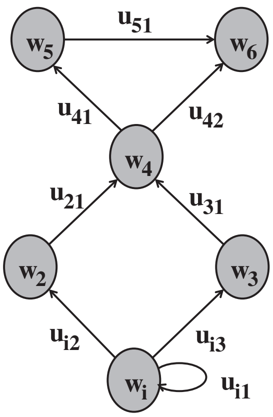

Fig. 2. State-transition diagram.

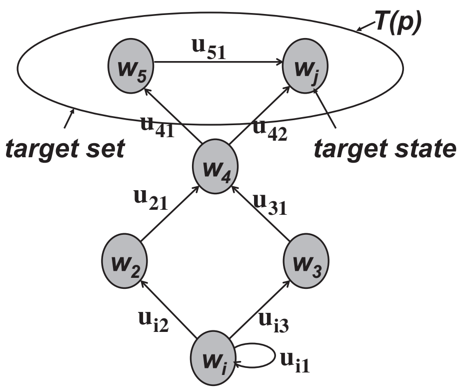

Fig. 3. Target set and target states. $T(p)$ is defined by p.

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