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A series of score functions for hesitant fuzzy sets

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ABSTRACT

This paper presents a series of score functions for hesitant fuzzy sets (HFSs) which provides us with a variety of new methods for ranking HFSs. The idea behind the methods is based on the score function for hesitant fuzzy elements (HFEs) that appears to be a subject worthy of further investigations in depth. The prominent characteristic of the HFS score-based method is that it can determine the priorities of the alternatives directly from the HFS score values. In a broad discussion, we describe a scheme for choosing an appropriate option from a set of HFS score-based methods according to what is expected from the score function and what is the nature of the values to be scored. Finally, the application of HFS ranking methods to multi-attribute decision making problems with hesitant fuzzy information is explained.

1. Introduction

Hesitant fuzzy multi-attribute decision making is a hot research topic which has recently received a great deal of attention from researchers [1,8,9,12,13]. Xia and Xu [9] developed a method to deal with multi-attribute decision making with anonymity based on a series of the hesitant fuzzy aggregation operators. Zhu et al. [13] proposed an approach to handle multi-attribute decision making problems using the weighted hesitant fuzzy geometric Bonferroni mean (WHFGBM) and the weighted hesitant fuzzy Choquet geometric Bonferroni mean (WHFCGBM) operators. Wei [8] investigated the hesitant fuzzy multi-attribute decision making problem in which the attributes are in different priority level. Wei [8] developed some prioritized aggregation operators to aggregate hesitant fuzzy information. Gu et al. [4] investigated the evaluation model for risk investment with hesitant fuzzy information and employed the hesitant fuzzy weighted averaging (HFWA) operator to aggregate the hesitant fuzzy information corresponding to each alternative. More recently, Farhadinia [2] developed an algorithm to do clustering under hesitant fuzzy environments by the use of the similarity measure of HFSs, and extended the algorithm to the case of interval-valued hesitant fuzzy sets (IVHFSs).

Ranking of alternatives plays an indispensable role in the hesitant fuzzy multi-attribute decision making problems. So far, two kinds of hesitant fuzzy ranking methods have been proposed [3,9] which are able to distinguish only the ingredients of hesitant fuzzy sets (HFSs), known as hesitant fuzzy elements (HFEs).

The main objective in this study is the determination of score value of HFSs that seems to be a missing discussion in the relevant literature. By deriving a series of interesting new score functions for HFSs, we then develop a HFS score-based method to solve hesitant fuzzy multi-attribute decision making problems. Compared to the existing methods [8,9,13], the HFS score-based method proceeds in less steps and its prominent characteristic is that it can relieve the laborious duty of using

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aggregation operators. Indeed, the HFS score-based method determines the priorities of the alternatives directly from the HFS score values.

The present paper is organized as follows: A series of new score functions for HFEs is presented in Section 2. In Section 3, it is indicated that the proposed score functions for HFEs meet some interesting properties. Section 4 is devoted to introducing a series of new score functions for HFSs based on the HFE score functions, and a practical example is brought to illustrate the applicability of the new score functions for HFSs. In Section 5, conclusion is drawn.

2. Preliminaries

This section is devoted to describing the basic definitions and notions of fuzzy set (FS) and its new generalization which are referred to as the hesitant fuzzy set (HFS).

An ordinary fuzzy set (FS) A in X is defined [11] as $A = \{\langle x, A(x)\rangle : x \in X\}$, where $A : X \to [0,1]$ and the real value $A(x)$ represents the degree of membership of x in A.

Definition 2.1 [7]. Let X be the universe of discourse. A hesitant fuzzy set (HFS) on X is symbolized by

$$H = \{\langle x, h(x)\rangle : x \in X\},$$

where $h(x)$, referred to as the hesitant fuzzy element (HFE), is a set of some values in [0, 1] denoting the possible membership degree of the element $x \in X$ to the set H.

Example 2.1. If $X = \{x_1, x_2, x_3\}$ is the discourse set, $h(x_1) = \{0.2, 0.4, 0.5\}, h(x_2) = \{0.3, 0.4\}$ and $h(x_3) = \{0.3, 0.2, 0.5, 0.6\}$ are the HFEs of $x_i (i = 1,2,3)$ to a set H, respectively. Then H can be considered as a HFS, i.e.,

$$H = \{\langle x_1, \{0.2, 0.4, 0.5\}\rangle, \langle x_2, \{0.3, 0.4\}\rangle, \langle x_3, \{0.3, 0.2, 0.5.0.6\}\rangle\}.$$

From a mathematical point of view, a HFS H can be seen as a FS if there is only one element in $h(x)$. In this situation, HFSs include FSs as a special case.

As will be shown later, the score function of the HFS is fundamentally defined in accordance with the score function of its HFEs. Here, we mainly discuss the score functions for HFEs and the corresponding score functions for HFSs can be obtained easily.

Hereafter, for notational convenience, h stands for the HFE $h(x)$ for $x \in X$, and we assume that $|h| = n$, that is,

$$h = \{h^{(1)}, h^{(2)}, \ldots, h^{(n)}\}.$$

In this contribution, we are looking for HFE score functions $S : [0,1]^n \to [0,1]$ which satisfy the following two properties:

1. Monotone non-decreasing property: if $h^{\sigma(1)} \leqslant \hat{h}^{\sigma(1)}, \ldots, h^{\sigma(n)} \leqslant \hat{h}^{\sigma(n)}$, then
$$S(h^{\sigma(1)}, \ldots, h^{\sigma(n)}) \leqslant S(\hat{h}^{\sigma(1)}, \ldots, \hat{h}^{\sigma(n)}), \tag{1}$$
where $h^{\sigma(j)}$ is referred to as the jth largest value in h whose elements are already arranged in increasing order. This rearrangement will be taken into account throughout the paper.
2. Boundary conditions property:
$$S(0, \ldots, 0) = 0, \quad \text{and} \quad S(1, \ldots, 1) = 1, \tag{2}$$
where we call $h = \{0\}$ and $h = \{1\}$ the empty HFE and the full HFE, respectively.

Now, we introduce a set of score functions for HFEs fulfilling the above properties.

Definition 2.2. Let $h = \{h^{(1)}, h^{(2)}, \ldots, h^{(n)}\}$ be a HFE. The following functions can be considered as the score functions for HFEs:

1. The smallest score function:
$$S_{\nabla}(h) = \begin{cases} 1, & \text{if } h \text{ is the full HFE, i.e., } h = \{1\}, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$
2. The greatest score function:
$$S_{\vartriangle}(h) = \begin{cases} 0, & \text{if } h \text{ is the empty HFE, i.e., } h = \{0\}, \\ 1, & \text{otherwise.} \end{cases} \tag{4}$$
3. The arithmetic-mean score function: