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Solving fuzzy complex system of linear equations

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ABSTRACT

This paper proposes a new and simple method for solving general fuzzy complex system of linear equations. In the original system, the elements of unknown variable vector and right hand side vector are considered as complex fuzzy number. Initially the general system is solved by adding and subtracting the left and right bounds of the fuzzy complex unknown and right hand side fuzzy complex vector respectively. Subsequently above obtained solutions are used to get the final solution of the general fuzzy complex system of linear equations. Two theorems are stated and proved related to the proposed method. A mathematical example and an application problem based on electrical circuit are solved using the present method. Results obtained are compared with the known results and are found to be in good agreement.

1. Introduction

Complex system of linear equations plays a vital role in real life problems such as optimisation, current flow, economics and engineering. A general complex system of linear equations may be written as $CZ = W$, where C and W are standard complex matrices and Z is the unknown complex vector. For the sake of simplicity, variables and parameters of these systems are defined exactly in the modelling. But in actual practice, the parameters and variables may be uncertain or vague and those are found, in general, by some experiment or experience. Hence, to overcome the uncertainty, one may use the complex numbers as fuzzy. Accordingly, fuzzy complex system of linear equations is considered as

$$[C]\{\widetilde{Z}\} = \{\widetilde{W}\},$$

where the coefficient matrix $[C]$ is an usual complex matrix, $\{\widetilde{W}\}$ is a column vector of fuzzy complex number and $\{\widetilde{Z}\}$ is the vector of fuzzy complex unknown.

In this regard, fuzzy and interval complex system of linear equations were investigated by various authors using different approaches. Fuzzy complex number was first proposed by Buckley [8]. Qiu et al. [29,30] discussed the sequence and series of fuzzy complex numbers and their convergence. Solution of fuzzy complex system of linear equations was described by Rahgooy et al. [31] and applied to circuit analysis problem. Jahantigh et al. [20] developed a numerical procedure for complex fuzzy linear systems. Recently, Behera and Chakraverty [6] proposed a new and simple centre and width based method for solving fuzzy real and complex system of linear equations. Solution sets of complex linear interval systems were investigated by Hladik [19]. Householder method was used by Djanybekov [14] for the solution of interval complex linear systems. In interval complex linear systems, the coefficient matrix was also taken as interval by the authors [14].

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Further, Candau et al. [9] analysed the complex interval arithmetic using polar form. Fuzzy modelling and identification procedure was implemented by Cao et al. [10] for the analysis and design of complex control systems. Filev [17] applied fuzzy approach to the control of nonlinear systems. Further Petkovic and Petkovic [28] investigated the complex interval arithmetic and applied it to various example problems. The authors [28], presented a circular form of the interval complex number. Complex interval arithmetic was also studied by Rokne and Lancaster [32]. Rump [33] developed software, viz. INTLAB, where the circular form of interval complex number was implemented.

On the other hand, fuzzy numbers are also implemented in the real systems. Friedman et al. [16] first proposed a general model to solve $n \times n$ fuzzy system of linear equations. Furthermore, many researchers [1–5,12,13,15,18,21–23,34–39] studied numerical and analytical solutions of real fuzzy system of linear equations. Behera and Chakraverty [7] recently investigated the solution procedure of fuzzy system of linear equations with polynomial parametric form of fuzzy numbers. Very recently, fuzzy centre and width based approach was used by Chakraverty and Behera [11] to solve fuzzy system of linear equations. Otadi and Mosleh [26] derived minimal solution of non-square fuzzy linear systems. Also, Otadi and Mosleh [27] provided fuzzy neural network approach for solving dual fully fuzzy linear systems. LU-factorization is extended to the fuzzy square matrix with respect to the max-product composition operator by Molai [24] to find the fuzzy lower and upper triangular matrices. Molai et al. [25] et al. proposed a new approach to solve a system of fuzzy polynomial equations based on the Gröbner basis.

This paper is organised as follows: Second section introduces preliminaries on fuzzy arithmetic and fuzzy complex arithmetic. Fuzzy complex system of linear equations with the proposed method are explained in Sections 3 and 4 respectively. In Section 5, numerical examples with plots of the results are incorporated to show efficiency of the proposed method. Last section includes the conclusion.

2. Preliminaries

In the following paragraph some terms related to the present work are defined [6,40].

Definition 2.1 (*Fuzzy number*). A fuzzy number U is a convex normalised fuzzy set U of the real line R such that

$$\{\mu_U(x) : R \to [0,1],\ \forall\, x \in R\},$$

where μ_U is called the membership function of the fuzzy set and is piecewise continuous.

Definition 2.2 (*Triangular fuzzy number (TFN)*). A triangular fuzzy number U is a convex normalised fuzzy set U of the real line R such that

i. There exists exactly one $x_0 \in R$ with $\mu_U(x_0) = 1$ (x_0 is called the mean value of U), where μ_U is called the membership function of the fuzzy set.
ii. $\mu_U(x)$ is piecewise continuous.

The membership function μ_U of an arbitrary triangular fuzzy number $U = (a, b, c)$ may be defined as follows:

$$\mu_U(x) = \begin{cases} 0, & x \leqslant a, \\ \frac{x-a}{b-a}, & a \leqslant x \leqslant b, \\ \frac{c-x}{c-b}, & b \leqslant x \leqslant c, \\ 0, & x \geqslant c. \end{cases}$$

Any arbitrary triangular fuzzy number $U = (a, b, c)$ can be represented with an ordered pair of functions through α-cut approach as $[\underline{u}(\alpha), \bar{u}(\alpha)] = [(b-a)\alpha + a,\ -(c-b)\alpha + c]$, where $\alpha \in [0,1]$. This satisfies the following requirements:

i. $\underline{u}(\alpha)$ is a bounded left continuous non-decreasing function over $[0,1]$.
ii. $\bar{u}(\alpha)$ is a bounded right continuous non-increasing function over $[0,1]$.
iii. $\underline{u}(\alpha) \leqslant \bar{u}(\alpha),\ 0 \leqslant \alpha \leqslant 1$.

Definition 2.3 (*Fuzzy arithmetic*). As discussed above, fuzzy numbers may be transformed into an interval through α-cut approach. So, for any arbitrary fuzzy number $x = [\underline{x}(\alpha), \bar{x}(\alpha)],\ y = [\underline{y}(\alpha), \bar{y}(\alpha)]$ and scalar k, we have the interval based fuzzy arithmetic as

i. $x = y$ if and only if $\underline{x}(\alpha) = \underline{y}(\alpha)$ and $\bar{x}(\alpha) = \bar{y}(\alpha)$,
ii. $x + y = [\underline{x}(\alpha) + \underline{y}(\alpha),\ \bar{x}(\alpha) + \bar{y}(\alpha)]$,
iii. $x - y = [\underline{x}(\alpha) - \bar{y}(\alpha), ,\ \bar{x}(\alpha) - \underline{y}(\alpha)]$,
iv. $x \times y = [\min(S), \max(S)]$, where $S = \{\underline{x}(\alpha) \times \underline{y}(\alpha),\ \underline{x}(\alpha) \times \bar{y}(\alpha),\ \bar{x}(\alpha) \times \underline{y}(\alpha),\ \bar{x}(\alpha) \times \bar{y}(\alpha)\}$,