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Analysis and extension of multiresolution singular value decomposition

Gaurav Bhatnagar [a,*], Ashirbani Saha [b], Q.M. Jonathan Wu [b], Pradeep K. Atrey [c]

[a] Indian Institute of Technology Jodhpur, Rajasthan, India
[b] University of Windsor, ON, Canada
[c] University of Winnipeg, MB, Canada

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ABSTRACT

This paper presents an analysis of the multiresolution form of singular value decomposition SVD (MR-SVD) and generalizes the analysis into a random multiresolution form of singular value decomposition (R-MR-SVD) with an intrinsic randomness. The core idea behind the proposed transform is to introduce randomness in the computing process based on parameters without which one can neither decompose nor reconstruct the data correctly. The proposed transform inherits the excellent properties of MR-SVD along with its own unique features, which can be useful in many research areas. Image encryption, lossy image compression, and face recognition are the primary applications used to illustrate the practical usage of the proposed transform.

1. Introduction

In the past decade, numerous mathematical transforms have been introduced to analyze signals. According to the requirements of signal processing applications, researchers have periodically modified these transforms. For instance, the basic tool for signal processing and analysis is the Fourier Transform (FT), which tries to approximate the signal in terms of sinusoids. One of the major drawbacks of FT is that it is not applicable to non-stationary signals. To overcome this problem, researchers have proposed several alternatives, namely short-time Fourier Transform, Gabor Transform, wavelet transform, etc.

Among these alternatives, the most frequently used and the most powerful is the wavelet transform (DWT). Several modifications have been performed on wavelets to make them useful in different scenarios. The core idea behind all these methods is to combine the multiresolution representation with filters depending on a threshold parameter. However, a conceptually complex structure, a huge computational memory (due to the use of filters), and limiting-perfect reconstruction (the term limiting indicates that the reconstruction is accurate to a specified degree) somewhat limit the use of DWT. On reviewing the wavelet literature, one finds that perfect reconstruction can only be achieved when the wavelet is applied on an infinite sequence of data. Almost all existing wavelets have the property of perfect reconstruction for infinite data sequences but do not exhibit the same property for finite data sequences. As a possible solution to these issues, Sweldens [21] and Kakarala and Ogunbona [12] devised the lifting wavelet transform (LWT) and multiresolution singular value decomposition (MR-SVD), respectively. Generally, it is difficult to find the lifting scheme for every existing wavelet easily.

* Corresponding author. Tel.: +91 9610170658.
E-mail address: goravb.uwindsor@gmail.com (G. Bhatnagar).

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Conversely, the MR-SVD scheme is very simple and has perfect reconstruction. Hence, this work concentrates on exploring MR-SVD and generalizing it to a random MR-SVD (R-MR-SVD).

In the past few years, some applications have been identified for MR-SVD. These applications include speaker recognition [14], speech signal enhancement [22], image compression [2,3], watermarking [1,13], stereo vision [15], fault diagnosis [23], image fusion [16] and other miscellaneous applications [10,19]. Among the possible applications of the newly proposed transform (R-MR-SVD), the primary and perhaps most important application lies in the area of information security. In information security, randomness plays a remarkable role, as does R-MR-SVD, as it introduces randomness to the process. To illustrate the practical applicability of R-MR-SVD, the proposed transform is applied to image encryption, lossy compression, and face recognition. Experimental results indicate that R-MR-SVD performs better performance than MR-SVD and DWT, which further demonstrates the possibilities of R-MR-SVD in information security. In compression, R-MR-SVD has similar performance when compared to MR-SVD and DWT at the lower compression ratios, and the performance enhances as we increase the compression ratio. R-MR-SVD and MR-SVD have similar performance levels, but perform better than DWT in face recognition.

The remainder of the paper is organized as follows: the MR-SVD is reviewed in Section 2 and the proposed R-MR-SVD is introduced in Section 3. The properties and implementation details of the proposed transform are described in Section 4. Three possible applications to image encryption, lossy image compression, and face recognition are discussed in Sections 5–7 respectively. Finally, the concluding remarks are given in Section 8.

2. Multiresolution singular value decomposition (MR-SVD)

The singular value decomposition (SVD) [11] of a real (imaginary) valued $M \times N$ matrix A, where $M \leqslant N$, is the factorization $A = USV^T$ where U and V are *orthogonal (unitary) matrices* and S is a *diagonal matrix* given by $S = diag(\sigma_1, \sigma_2, \ldots, \sigma_r)$, where $\sigma_i, i = 1(1)r$ are the singular values of matrix A with $r \leqslant \min(M, N) = M$ and satisfying $\sigma_1 \geqslant \sigma_2 \geqslant \cdots \geqslant \sigma_r$. The first r columns of V are called *right singular vectors* and the first r columns of U are called *left singular vectors*. Motivated by the excellent properties of SVD, Kakarala generalized SVD to MR-SVD in 2001. The basic idea of their extension is to replace low and high pass filters (used in wavelet transform) with SVD. The detailed flow of MR-SVD is depicted in Fig. 1 and is described as follows:

Let $X = [x(i,j)]_{i \in [1,m], j \in [1,n]}$ represent a 2D finite signal of size $m \times n$ such that m and n are dyadic. The computation of MR-SVD corresponds to the following steps,

1. Segment X into non-overlapping blocks of size $p \times q$ and arrange each block into a $pq \times 1$ vector by stacking columns to form the matrix X_1 of size $pq \times mn/pq$.
2. Evaluate the corresponding centered matrix i.e. $\overline{X}_1$ followed by the scatter matrix (T_1) computation as $T_1 = \overline{X}_1 \overline{X}_1^T$ of size $pq \times pq$.
3. Let U_1 be the modal matrix (matrix of eigenvectors) which diagonalize the scatter matrix T_1, i.e., $U_1^T T_1 U_1 = S_1^2$, where $S_1^2 = diag\left(s_1(1)^2, s_1(2)^2, \ldots, s_1(pq)^2\right)$ contains the squares of the singular values of T_1.
4. Construct matrix $\widehat{X}$ as $\widehat{X} = U_1^T \overline{X}_1$.

Approximation and detail parts can be obtained by stacking the rows of $\widehat{X}$ into an array. The first row of $\widehat{X}$ corresponds to the largest singular value and is hence considered the approximate part of the signal. Similarly, the remaining row of $\widehat{X}$ corresponds to the remaining singular values (in decreasing order) and is hence considered the detail parts of the signal i.e. $\phi_1 = \widehat{X}(1,:)$ and $\psi_1^i = \widehat{X}(i,:)$, where ψ_1^i is the ith detail sub-band at level 1.

3. Random multiresolution singular value decomposition (R-MR-SVD)

This section defines a random form of multiresolution SVD (R-MR-SVD). The R-MR-SVD can be derived from the essence of the MR-SVD. Since, R-MR-SVD is the generalization of the MR-SVD, it satisfies all the properties of MR-SVD. The core idea behind the generalization from MR-SVD to R-MR-SVD is to produce randomness in the process. Randomness is produced by

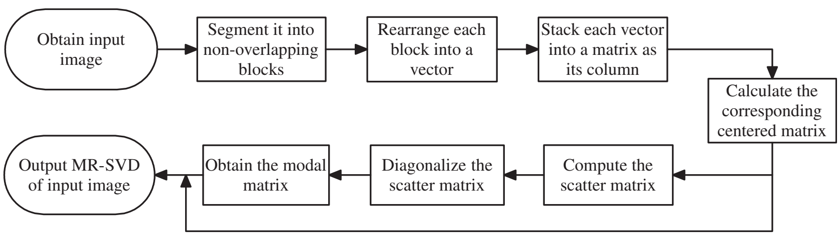

Fig. 1. Working flow of MR-SVD.

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