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Fuzzy inferior ratio method for multiple attribute decision making problems

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ABSTRACT

Multiple attribute decision making forms an important part of the decision process for both small (individual) and large (organization) problems. When available information is precise, many methods exist to solve this problem. But the uncertainty and fuzziness inherent in the structure of information make rigorous mathematical models inappropriate for solving this type of problems. This paper incorporates the fuzzy set theory and the basic nature of subjectivity due to the ambiguity to achieve a flexible decision approach suitable for uncertain and fuzzy environment. The proposed method can take both real and fuzzy inputs. An outranking intensity is introduced to determine the degree of overall outranking between competing alternatives, which are represented by fuzzy numbers. Numerical examples finally illustrate the approach.

1. Introduction

Multiple attribute decision-making (MADM) methods are widely used to rank real world alternatives or select the best alternative with respect to several competing criteria. In classical MADM methods, assessments of alternatives are precisely known [3,30,32]. Due to fuzziness and uncertainty of decision-making problems, and the inherent vagueness of human preferences, however, the best expression of decision makers comes in natural language. As a result, using linguistic (fuzzy) assessments are much more realistic than numerical values. In other words, linguistic variables, in which the values are words or sentences from natural or artificial languages, are used in the assessment of alternatives with respect to criteria [18,29,33,38,39].

An MADM problem can be concisely expressed in matrix format as

$$\boldsymbol{A} = \begin{matrix} & c_1 & c_2 & \cdots & c_m \\ A_1 & a_{11} & a_{12} & \cdots & a_{1m} \\ A_2 & a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_n & a_{n1} & a_{n2} & \cdots & a_{nm} \end{matrix} \tag{1.1}$$

where $A_1, \ldots, A_n$ are feasible alternatives among which decision should be made, $c_1, \ldots, c_m$ are attributes with which the performance of alternatives are measured, and the entry a_{ij} of decision matrix $\boldsymbol{A}$ is the rating of alternative A_i with respect to the

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attribute c_j. Furthermore, the attributes normalized weight vector $\boldsymbol{w} = (w_1, \ldots, w_m)^t$ should be either appraised by pairwise comparisons or determined by decision maker's (DM) preferences [19,23,40].

Assume that the alternatives set is denoted by Λ and the attributes set is denoted by Υ; that is, $\Lambda = \{A_1, \ldots, A_n\}$ and $\Upsilon = \{c_1, \ldots, c_m\}$. In general terms, attributes are divided into benefit attributes and cost attributes. That is to say, Υ is partitioned into two distinct sets Υ^+ and Υ^-. Then, an MADM problem can be portrayed as follows [16,28,35]:

$$\begin{aligned} &\text{Maximize} \quad \{a_{ij} : j \in \Upsilon^+\} \\ &\text{Minimize} \quad \{a_{ij} : j \in \Upsilon^-\} \\ &\text{Subject to} \quad A_i \in \Lambda \end{aligned} \tag{1.2}$$

In classical MADM methods, the attribute values (ratings) and weights are determined precisely. A survey of the methods has been presented in [17]. In real life management decision situations, MADM models and methods have been proposed. So far, a variety of applicable methods such as Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) and Analytic Hierarchy Process (AHP) have been developed for an MADM problem.

TOPSIS was developed for solving MADM problems based on the concept that the obtained solution should be nearest to the positive ideal solution (PIS) and be remotest from the negative ideal solution (NIS). TOPSIS is a popular method and has been widely used in the literature (Abo-Sinna and Amer [1]; Agrawal et al. [2]; Cheng et al. [9]; Mokhtarian and Hadi-Vencheh [31]; Deng et al. [13]; Feng and Wang [14,15]; Chen and Liu [7]; Hwang and Yoon [19]; Jee and Kang [20]; Kim et al. [24]; Li [27]). The method has also been extended to deal with fuzzy MADM problems. For example, Tsaur et al. [36] first convert a fuzzy MADM problem into a real one via centroid defuzzification and then solve the non-fuzzy MADM problem using the TOPSIS method. Chen and Tzeng [8] transform a fuzzy MADM problem into a non-fuzzy MADM using fuzzy integral. Instead of using distance, they employ gray relation grade to define the relative closeness of each alternative. Chu [10,11] and Chu and Lin [12] also change a fuzzy MADM problem into a real one and solve the real MADM problem using the TOPSIS method. Differing from the others, they first derive the membership functions of all the weighted ratings in a weighted normalization decision matrix using interval arithmetics of fuzzy numbers and then defuzzify them into real values using the ranking method of mean of removals [22]. Chen [6] extends the TOPSIS method to fuzzy group decision making situations by defining a real Euclidean distance between any two fuzzy numbers. Triantaphyllou and Lin [35] develop a fuzzy version of the TOPSIS method based on fuzzy arithmetic operations, which leads to a fuzzy relative closeness for each alternative. It is argued that fuzzy weights and fuzzy ratings should result in fuzzy relative closeness. Real relative closeness provides only one possible solution to a fuzzy MADM problem, but cannot reflect the whole picture of its all possible solutions. Wang and Elhang [37] propose a fuzzy TOPSIS method based on alpha level sets. They deal with the relative closeness as an optimal solution to a fractional programming.

In all aforementioned works, the authors develop a hybrid fuzzy TOPSIS. TOPSIS, as will be seen in next section, seeks for a compromise solution (alternative) that is closest to ideal solution and remotest from negative ideal solution. However, the compromise solution of TOPSIS is not essentially the remotest from negative ideal solution. Therefore, the ranking coefficients concern only to closeness to the ideal solution (see Example 1 in sub-Section 2.1). Motivated by such a fact, this paper proposes a method for solving fuzzy MADM.

In this paper, we develop a new approach for solving multiple attribute decision making problems based on the same concept as TOPSIS considering both distances to the PIS and from the NIS. Furthermore, constructing the model under the assumption of vagueness and imprecise environment enables the method to conform to real life decision situations.

This paper is organized as follows. In the following section the essential definitions and concepts, as well as, notations of the fuzzy set theory are exhibited. Section 3 is dedicated to the proposed method and its hints. Section 4 illustrates the proposed method with two numerical examples. Ultimately, discussion and conclusion are given in Section 5.

2. Preliminaries

Fuzzy sets are coherent extension of real sets and were first developed by Zadeh [41] as an aid for dealing with uncertainty/imprecision and vagueness in the real world. A fuzzy set is a collection of elements in a universe of information where the boundary of the set contained in the universe is ambiguous, vague and fuzzy. Each fuzzy set is specified by a membership function, which assigns to each element in the universe of discourse a value within the unit interval $[0, 1]$. The assigned value is called degree (or grade) of membership, which specifies the extent to which a given element belongs to the fuzzy set or is related to a concept. If the assigned value is 0, then the given element does not belong to the set. If the assigned value is 1, then the element totally belongs to the set. If the value lies within the interval $(0, 1)$, then the element only partially belongs to the set. Therefore, any fuzzy set can be uniquely determined by its membership function.

Let X be the universe of discourse. A fuzzy set $\widetilde{A}$ of the universe of discourse X is said to be convex if and only if for all x_1 and x_2 in X there always exists $\lambda \in [0, 1]$ such that:

$$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geqslant \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \tag{2.1}$$

where $\mu_{\tilde{A}}$ is the membership function of the fuzzy set $\widetilde{A}$. A fuzzy set $\widetilde{A}$ of the universe of discourse X is said to be normal if there exists a $x_i \in X$ satisfying $\mu_{\tilde{A}}(x_i) = 1$. Fuzzy numbers are special cases of fuzzy sets that are both convex and normal. The membership function of a fuzzy number is piecewise continuous and satisfies the following properties: