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Intuitionistic fuzzy multigranulation rough sets

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ABSTRACT

Exploring rough sets from the perspective of multigranulation represents a promising direction in rough set theory, where concepts are approximated by multiple granular structures represented by binary relations. Through a combination of multigranulation rough sets with intuitionistic fuzzy rough sets, this study develops a new multigranulation rough set model, called an intuitionistic fuzzy multigranulation rough set (IFMGRS). In the multigranulation framework, three types of IFMGRSs that are generalizations of three existing intuitionistic fuzzy rough set models are proposed. First, we present three types of IFMGRSs. From their basic properties, we conclude that they are extensions of three existing intuitionistic fuzzy rough sets. Second, we define the reducts of the three types of IFMGRSs to eliminate redundant intuitionistic fuzzy granulations. Third, we examine the reduction approaches of IFMGRS with a detailed example and discuss the general reduction theory of IFMGRS.

1. Introduction

It is well known that rough set theory [32] was introduced by Pawlak as a valid means of granular computing [33]. The basic tools are relations that represent information systems or decision tables. In Pawlak's rough set theory, a relation is an equivalence where the relation may be a dominance, covering, similarity, tolerance, fuzzy, neighbor, or other indiscernibility one within various generalized rough set models [1,3,10–12,15,17,18,24,29,30,41,48,53,55,58,62].

Intuitionistic fuzzy (IF) set theory was introduced by Atanassov [4,5] as an intuitively straightforward extension of fuzzy set theory. It has been successfully applied in many fields for decision analysis and pattern recognition [7,9,42–44,46]. Rough sets and IF sets both capture particular facets of the same notion-imprecision. Studies of the combination of IF set theory and rough set theory is being acknowledged as a positive approach to rough set theory. Recently, Zhou et al. [59,61] examine IF rough set approximation operators where both constructive and axiomatic approaches are used, and the characterizations of rough set approximations in IF set theory are studied in [60]. Zhang [56] has researched generalized IF rough sets based on IF coverings. Huang et al. [19–21] discuss dominance-based IF and interval-valued IF rough set models and their applications. Zhang et al. [54] provide a systematic study of a general framework of IF rough sets.

One important characteristic of various rough set models is that a target concept is always characterized by the so-called upper and lower approximations under a single granulation, i.e., the concept is depicted by available knowledge induced from a single relation on the universe. From the perspective of granular computing, an equivalence (or a tolerance, similarity,

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covering, dominance, fuzzy, IF) relation on the universe can be regarded as a granulation, and a partition (or a family of sets) determined by its relation on the universe becomes a granulation space [40]. We can thus conclude that a number of rough set models are based on a single granulation. However, as illustrated in [38], in many cases it is more reasonable to describe the target concept through multiple relations on the universe according to user requirements or problem solving targets. To apply rough set theory in practical applications more widely, Qian and Liang [38] extend Pawlak's single-granulation rough set model to a multigranulation rough set (MGRS) model. In Qian's MGRS model, two different basic models are defined: one is the optimistic MGRS, the other is the pessimistic MGRS. At Present, rapid developments have been achieved in MGRS area. For instance, Qian et al. [36] extend the rough set model based on a tolerance relation to an incomplete rough set model based on multigranulations, where set approximations are defined through multiple tolerance relations on the universe. Abu-Donia [2] studies rough approximations through a multi-knowledge base. Wu and Leung [45] investigate a multi-scale information system that explains a problem using different scales (levels of granulation). She and He [39] discuss fundamental properties of the MGRS model. In order to extend the theory of MGRS, Lin et al. [27,28] develop neighborhood-based and covering-based MGRS. Liang et al. [26] propose an efficient rough feature selection algorithm for large-scale data sets inspired by multigranulation. Xu et al. [47] propose a new fuzzy MGRS based on tolerance relations. Yang et al. [50] generalize the MGRS model into fuzzy environment.

Although both IF rough sets and MGRS are two important generalizations of the classical rough set model, there are few studies on the combination of IF set theory and MGRS. In this paper, we examine the intuitionistic fuzzy multigranulation rough set (IFMGRS), and we focus on three types of intuitionistic fuzzy multigranulation rough sets.

The rest of this paper is organized as follows. Section 2 briefly introduces the preliminary concepts considered in the study, such as IF sets, three types of IF rough sets, MGRS, fuzzy MGRS, and so on. Section 3 presents three types of IFMGRSs, that is, Type-I, Type-II, and Type-III IFMGRSs; we also discuss their properties and conclude that they are extensions of three existing IF rough sets. In Section 4, we propose the reducts of these IFMGRSs to eliminate redundant IF granulations. In Section 5, we examine the reduction approaches of Type-I IFMGRS with a detailed example. Finally, Section 6 concludes this paper.

2. Bacic concepts

In this section, we recall some basic concepts, such as IF set, IF relation, IF rough sets, MGRSs, and so on.

Definition 2.1 (*[4,5]*). Given the universe of discourse U, an IF set (IFS) A in U is an object having the form $A=\{<x,\mu_A(x),\gamma_A(x)>|x\in U\}=\sum_{x\in U}\frac{\langle\mu_A(x),\gamma_A(x)\rangle}{x}$, where $\mu_A:U\to[0,1]$ and $\gamma_A:U\to[0,1]$ satisfy $0\leqslant\mu_A(x)+\gamma_A(x)\leqslant 1$ for all $x\in U$; $\mu_A(x)$ and $\gamma_A(x)$ are, respectively, called the degree of membership and the degree of nonmembership of the element $x\in U$ to A. Furthermore, $\pi_A(x)=1-\mu_A(x)-\gamma_A(x)$ is called the degree of hesitancy of $x\in U$ to A. The family of all IF sets in U is denoted by $IF(U)$.

Definition 2.2 (*[4,5]*). Let $A=\{<x,\mu_A(x),\gamma_A(x)>|x\in U\}, B=\{<x,\mu_B(x),\gamma_B(x)>|x\in U\}\in IF(U)$, then

(1) The supplementary set of A, $\sim A=\{<x,\gamma_A(x),\mu_A(x)>|x\in U\}$;
(2) $A=B\Longleftrightarrow\mu_A(x)=\mu_B(x)\wedge\gamma_A(x)=\gamma_B(x)$;
(3) $A\subseteq B\Longleftrightarrow\mu_A(x)\leqslant\mu_B(x)\wedge\gamma_A(x)\geqslant\gamma_B(x)$;
(4) $A\cap B=\{<x,\min\{\mu_A(x),\mu_B(x)\},\max\{\gamma_A(x),\gamma_B(x)\}>\}$;
(5) $A\cup B=\{<x,\max\{\mu_A(x),\mu_B(x)\},\min\{\gamma_A(x),\gamma_B(x)\}>\}$;
(6) $A-B=A\cap\sim B$.

Definition 2.3 [6]. An IF relation $\mathscr{R}$ on U is an IFS in $U\times U$, i.e., $\mathscr{R}$ is given by

$$\mathscr{R}=\{<(x,y),\mu_{\mathscr{R}}(x,y),\gamma_{\mathscr{R}}(x,y)>|(x,y)\in U\times U\},$$

where $\mu_{\mathscr{R}}:U\times U\to[0,1]$ and $\gamma_{\mathscr{R}}:U\times U\to[0,1]$ satisfy $0\leqslant\mu_{\mathscr{R}}(x,y)+\gamma_{\mathscr{R}}(x,y)\leqslant 1$ for all $(x,y)\in U\times U$.

Let $\mathscr{R}$ be an IF relation on U, $\mathscr{R}$ is reflexive if $\mu_{\mathscr{R}}(x,x)=1$ and $\gamma_{\mathscr{R}}(x,x)=0$ for all $x\in U$; $\mathscr{R}$ is symmetric if $\mu_{\mathscr{R}}(x,y)=\mu_{\mathscr{R}}(y,x)$ and $\gamma_{\mathscr{R}}(x,y)=\gamma_{\mathscr{R}}(y,x)$ for all $(x,y)\in U\times U$; $\mathscr{R}$ is transitive if for all $(x,z)\in U\times U$, $\mu_{\mathscr{R}}(x,z)\geqslant\bigvee_{y\in U}[\mu_{\mathscr{R}}(x,y)\wedge\mu_{\mathscr{R}}(y,z)]$ and $\gamma_{\mathscr{R}}(x,z)\leqslant\bigwedge_{y\in U}[\gamma_{\mathscr{R}}(x,y)\vee\gamma_{\mathscr{R}}(y,z)]$. $\mathscr{R}$ is transitive if and only if the following conditions are satisfied: for all $x,y,z\in U$, $\alpha_1,\alpha_2\in[0,1]$, $\mu_{\mathscr{R}}(x,y)\geqslant\alpha_1$ and $\mu_{\mathscr{R}}(y,z)\geqslant\alpha_1$ imply $\mu_{\mathscr{R}}(x,z)\geqslant\alpha_1$; $\gamma_{\mathscr{R}}(x,y)\leqslant\alpha_2$ and $\gamma_{\mathscr{R}}(y,z)\leqslant\alpha_2$ imply $\gamma_{\mathscr{R}}(x,z)\leqslant\alpha_2$.

Similar to classical binary relations, IF relations on a finite universe can be denoted by an IF matrix.

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