ARTICLE IN PRESS

Information Sciences xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Analysis and synthesis of randomly switched systems with known sojourn probabilities

Engang Tian [a], Dong Yue [b,c,*], Tai-cheng Yang [d]

[a] School of Electrical and Automation Engineering, Nanjing Normal University, 78 Bancang Street, Nanjing, Jiangsu 210042, China
[b] College of Automation Engineering, Nanjing University of Posts and Telecommunications, Nanjing, China
[c] Department of Control Science and Engineering, Huazhong University of Science and Technology, Wuhan 430074, China
[d] Department of Engineering and Design, University of Sussex, Brighton, BN1 9QT England, United Kingdom

ARTICLE INFO

Article history:
Received 21 November 2012
Received in revised form 2 August 2013
Accepted 18 February 2014
Available online xxxx

Keywords:
Randomly switched system
Known sojourn probability
Switching law
Lyapunov functional method

ABSTRACT

In this paper, a new approach is proposed and investigated for the stability analysis and stabilizing controller design of randomly switched linear discrete systems. The approach is based on sojourn probabilities and it is assumed that these probabilities are known a prior. A new Lyapunov functional is constructed and two main theorems are proved in this paper. Theorem 1 gives a sufficient condition for a switched system with known sojourn probabilities to be mean square stable. Theorem 2 gives a sufficient condition for the design of a stabilizing controller. The applications of these theorems and the corresponding corollary and lemma are demonstrated by three numerical examples. Finally, some future research is proposed.

1. Introduction

Many systems and controls show dynamics of a switched system: power systems, networked control systems, traffic control, electronics control, etc. [1,9,10,17,18,21]. Switched system is also an interesting and challenging area in theory. For example, even when all the subsystems are exponentially stable, the switched systems may have divergent trajectories for certain switching signals. Another remarkable fact is that one may carefully switch between unstable subsystems to make the switched system exponentially stable. There is a great deal of research on the switching and switched systems. First, switching between subsystems can be abrupt or arbitrarily (Section 2 of the survey paper [10] gives a good review of the stability conditions for arbitrarily switched systems). Secondly, in contrast to arbitrarily switching, there is a class of switched systems where the switching among different subsystems can be completely defined by the trajectory of the system state variables (Interested readers may refer to a brief overview presented in the introduction of a recent paper [13] and the references therein). Thirdly, the stability analysis can be based on the knowledge of the minimum dwell time or average dwell time of the system. This leads to a dwell-time or average dwell-time based approach (see Section 3 of [10] for a survey). Fourthly, there is a Markovian jump system based approach. Recently we have seen a significant number of publications based on this approach [11,29,31,36–38]. A switched linear system is modeled as a Markovian Jump System (MJS) where the switching is governed by a Markov process. For example, packet dropouts and channel communication

* Corresponding author at: College of Automation Engineering, Nanjing University of Post and Telecommunications, Nanjing, China.
E-mail addresses: teg@njnu.edu.cn (E. Tian), medongy@vip.163.com (D. Yue), t.c.yang@sussex.ac.uk (T.-C. Yang).

http://dx.doi.org/10.1016/j.ins.2014.02.129

delays in networked control systems can be modeled by Markov Chains [6,16,20,28,32]. To apply this approach, it is necessary to have a completely known or a partially known Transition Probability Matrix (TPM).

In this paper, a new approach for the stability analysis and stabilizer design is developed for switched linear discrete systems. This approach is based on known sojourn probabilities (KSP). A sojourn probability of a subsystem is the probability of a switched system staying in that subsystem. Sojourn probabilities can be easily obtained from the history of the system. In the study of switched systems, there is a main difficulty in applying the Markovian jump system approach. Transition probability matrices required in the MJS approach are hard or costly to obtain [36]. For a switched system consists of n subsystems, any element in an n by n TPM is the probability of the system switching from the i-th subsystem to the j-th subsystem. On the other hand, sojourn probabilities for the same system are only a set of n numbers. These numbers are not related to any details of the switching. This motivates our research on a new KSP based approach.

The rest of this paper is organized as follows. A new model of a switched system with KSP is formulated in Section 2.1. It is shown that many practical problems can be represented by this model. In Section 2.2, the condition of the mean square stability of the formulated switched system is developed (Theorem 1). In Section 2.3, Theorem 2 gives the condition for the controller design. Three numerical examples are shown in Section 3 to demonstrate the usage and the advantages of the proposed approach. The paper ends with a short conclusion and a summary of five possible directions/topics for further research.

2. Stability analysis and stabilizing controller design based on the KSP

2.1. Problem formulation

Consider a discrete-time switched system with delays:

$$x(k+1) = A_{\sigma(k)}x(k) + A_{d\sigma(k)}x(k - d_{\sigma(k)}(k)) + B_{\sigma(k)}u(k) \tag{1}$$

where $x(k) \in Z^+ \to \mathbb{R}^n$ is the state vector, $u(k) \in Z^+ \to \mathbb{R}^m$ is the control input, $\sigma(k) : Z^+ = \{0, 1, 2, \ldots\} \to \{1, 2, \ldots, N\} \triangleq \Omega$ is the switching actions independent of the state. A_i, A_{di}, B_i $(i \in \Omega)$ are matrices with compatible dimensions for the ith subsystem, $d_i(k)$ is the time-varying delay of the ith subsystem satisfying $d_i^m \leqslant d_i(k) \leqslant d_i^M$. The controller is

$$u(k) = K_{\sigma(k)}x(k) \tag{2}$$

where $K_i(i \in \Omega)$ is the controller feedback gain of the ith subsystem to be designed. Substitute (2) into (1)

$$x(k+1) = \left(A_{\sigma(k)} + B_{\sigma(k)}K_{\sigma(k)}\right)x(k) + A_{d\sigma(k)}x(k - d_{\sigma(k)}(k)) \tag{3}$$

In this paper, the probability of $\sigma(k) = i$ is assumed to be known, i.e.,

$$\Pr\{\sigma(k) = i\} = \alpha_i, \quad \sum_{i=1}^{N}\alpha_i = 1 \tag{4}$$

where $\alpha_i \in (0, 1)$ is the sojourn probability of the switched system staying in the ith subsystem. It is not difficult to obtain the statistic information α_i through simple statistical way.

$$\alpha_i = \lim_{k\to\infty}\frac{k_i}{k}$$

where k_i is the times of $\sigma(k) = i$ in the interval $[1, k], k \in Z^+$.

A set of random variables $\gamma_i(k) : Z^+ \to \{0, 1\}$ is used:

$$\gamma_i(k) = \begin{cases} 1, & \sigma(k) = i \\ 0, & \sigma(k) \neq i \end{cases}, \quad i \in \Omega, \quad k \in Z^+, \tag{5}$$

and for any $k \in Z^+$

$$\sum_{i=1}^{N}\gamma_i(k) = 1, \quad \mathbb{E}\{\gamma_i(k)\} = \alpha_i, \quad \sum_{i=1}^{N}\alpha_i = 1. \tag{6}$$

The first equation in (6) is to guarantee that there is only one active subsystem at any time. Based on (3)–(6), the switched system can be rewritten as

$$x(k+1) = \sum_{i=1}^{N}\gamma_i(k)\{(A_i + B_iK_i)x(k) + A_{di}x(k - d_i(k))\} \tag{7}$$

$$x(k) = \phi(k), \quad k = -d^M, \quad -d^M + 1, \ldots, 0 \tag{8}$$

where $d^M = \max\{d_i^M, i \in \Omega\}, \phi(k)$ is the initial state of $x(k)$.

Please cite this article in press as: E. Tian et al., Analysis and synthesis of randomly switched systems with known sojourn probabilities, Inform. Sci. (2014), http://dx.doi.org/10.1016/j.ins.2014.02.129