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Variable consistency dominance-based rough set approach to preference learning in multicriteria ranking

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A B S T R A C T

We present a methodology for non-statistical preference learning in multicriteria ranking based on Variable Consistency Dominance-based Rough Set Approach (VC-DRSA). A finite set of objects to be ranked is evaluated by a set of criteria, which are real-valued functions with ordinal or cardinal scales. Given the statement of a multicriteria ranking problem, the only objective information one can get is the dominance relation over the set of objects. The dominance relation is, however, too poor because it leaves many objects incomparable. To enrich this relation, and make the objects more comparable, a decision maker (DM) must supply some preference information revealing her/his value system with respect to multicriteria evaluations. We are considering a frequent case, when the preference information has the form of pairwise comparisons of some objects relatively well known to the DM, called reference objects. This information is thus composed of some decision examples on the reference objects. It is the input data for a method that learns the preferences of the DM. Since this information is prone to inconsistencies, we propose to structure it using VC-DRSA. Then, the pairs of objects that are sufficiently consistent serve as a basis for induction of a preference model. This model has the form of a set of "*if* . . ., *then* . . ." decision rules. Application of these rules on the whole set of objects to be ranked yields a fuzzy preference structure (directed weighted graph). This preference structure is then exploited using a ranking method, so as to work out a final recommendation. We propose a list of properties that helps to choose a proper ranking method. The methodology is illustrated by an example.

1. Introduction

1.1. Ranking problem as a problem of multicriteria decision aiding and preference learning

In this paper, we present a methodology for dealing with a *multicriteria ranking problem* using a preference model in the form of a set of decision rules induced from decision examples. The ranking consists in ordering a set of *objects* (also called alternatives, solutions, acts, actions, options, candidates, . . .) from the best to the worst, while these objects are evaluated from multiple points of view considered relevant for the problem at hand and called *criteria* (also called attributes, features,

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variables, ...). Multicriteria ranking problems constitute one of three main categories of decision problems considered in the field of *Multiple Criteria Decision Aiding* (also called *Multiple Criteria Decision Making*), which are ranking, choice and sorting (also called ordinal classification). As pointed out by Dembczyński et al. [28], Multicriteria Decision Aiding shares some goals, concepts and methodological issues with *Preference Learning* being an emerging subfield of Machine Learning. The main difference between them consists in the way of building a preference model of the Decision Maker (DM). While in Preference Learning (PL) the preference model results from statistical analysis of data (training examples), in Multicriteria Decision Aiding (MCDA) it is built from preference information elicited from the DM, very often interactively.

An important step in MCDA concerns selection and construction of criteria used for evaluation of objects. They are real-valued functions with ordinal or cardinal scales, built on elementary features of the objects. The aim is to set up a set of criteria which makes the pairwise comparison of all objects in the considered set meaningful. The criteria are equipped with monotonic preference scales which specify the preference orders in their value sets.

Remark that while in MCDA the construction of criteria with explicit monotonic preference scales is an important step in the procedure of decision aiding, in PL, the relationships between value sets of attributes and DM's preferences are discovered from data for a direct use in decision making. This means that in PL, the monotonic preference scales converting elementary features to criteria are neither used nor revealed explicitly.

For a given finite set of objects A, and for a finite set of criteria $G = \{g_1, \ldots, g_n\}$ giving evaluations $g_i(a)$ to all $a \in A, i = 1, \ldots, n$, the only objective information that comes out from comparison of these objects on multiple criteria is a *dominance relation D* over set A. Given $a, b \in A$, object a dominates object b, which is denoted by aDb, if and only if $a \succeq_i b$ for each $i = 1, \ldots, n$, where $a \succeq_i b$ means that $g_i(a)$ "is at least as good as" $g_i(b)$. As $\succeq_i$ is a complete preorder over A, dominance relation D is a partial preorder, i.e., a reflexive and transitive binary relation defined over A.

Apart from trivial cases, dominance relation D is rather poor and leaves many objects incomparable (objects $a, b \in A$ are incomparable if neither aDb nor bDa). In order to enrich the dominance relation and make the objects in A more comparable, one needs additional information about value system of the DM, called *preference information*. This information permits to build a more or less explicit model of DM's preferences, called *preference model*. The preference model relates the decision to evaluations of the objects on the considered criteria. In other words, the preference model aggregates multicriteria evaluations of objects. It is inducing a *preference structure* on set A. A proper exploitation of this structure leads then to a *recommendation* in terms of ranking of objects from set A.

In PL, the training data are the equivalent of preference information in MCDA. Moreover, the aim of getting a preference model which permits to work out a final recommendation is the same for both methodologies – roughly speaking, the difference resides in statistical or non-statistical way of processing the preference information.

It follows from above that the preference information and the preference model are two crucial components of both MCDA and PL. The many methods existing in both fields differ just by these two components. Below, with respect to these two components, we review some recent trends in MCDA.

1.2. Preference information and preference model

As to the preference information, it depends on the adopted methodology: prices and interest rates for cost-benefit analysis, cost coefficients in objectives and technological coefficients in constraints for mathematical programming, a training set of decision examples for neural networks and machine learning, substitution rates for a value function of Multi-Attribute Utility Theory, pairwise comparisons of objects in terms of intensity of preference for the Analytic Hierarchy Process, attribute weights and several thresholds for ELECTRE methods, and so on (see the state-of-the-art survey by Figueira et al. [31]). This information has to be provided by the DM, possibly assisted by an analyst.

Very often the preference information is not easily definable. For example, this is the case of the price of many immaterial goods and of the interest rates in cost-benefit analysis, or the case of the coefficients of objectives and constraints in mathematical programming models. Even if the required information is easily definable, like a training set of decision examples for neural networks, it is often processed in a way which is not clear for the DM, such that (s)he cannot see what are the exact relations between the provided information and the final recommendation. Consequently, very often the decision aiding method is perceived by the DM as a *black box* whose result has to be accepted because the analyst's authority guarantees that the result is "right". In this context, the aspiration of the DM to find good reasons to make decision is frustrated and raises the need for a more transparent methodology in which the relation between the original information and the final recommendation is clearly shown. Such a transparent methodology searched for has been called *glass box* [51]. Its typical representative is using a training set of decision examples as the input preference information.

The decision examples may either by provided by the DM on a set of real or hypothetical objects, or may come from observation of DM's past decisions. Such an approach follows the paradigm of inductive learning used in artificial intelligence [65], or robust ordinal regression becoming popular in operational research [56]. It is also concordant with the principle of posterior rationality postulated by March [64] since it emphasizes the discovery of DM's intentions as an interpretation of actions rather than as a priori position. This paradigm has been used to construct various preference models from decision examples, e.g., the general additive utility functions [32,54], the outranking relations [41,66], the monotonic decision trees [40], and the set of "*if* ..., *then* ..." decision rules [48].

Of particular interest is the last model based on decision rules – it has been introduced to decision analysis by Greco, Matarazzo and Słowiński [44,46,85]. A popular saying attributed to Slovic [81] is that "people make decisions and then