Information Sciences 276 (2014) 31–41

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Classification of coverings in the finite approximation spaces

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ARTICLE INFO

Article history:
Received 16 December 2011
Received in revised form 16 January 2014
Accepted 9 February 2014
Available online 21 February 2014

Keywords:
Covering rough set
Neighborhood
Complementary neighborhood
Topology

ABSTRACT

Based on the notions of neighborhood and complementary neighborhood, we consider the classification of coverings in the covering rough set theory. We present a classification rule under which the coverings on a universe U are classified such that for any given pair of neighborhood (or complementary neighborhood)-based lower and upper approximation operators on U, all the different coverings in the same class generate the same pair of lower and upper approximations of each $X \subset U$. We show that there is a one to one correspondence between the equivalence classes of coverings and the topologies of the universe. Thus the number of the equivalence classes of coverings is equal to the number of topologies of the universe, and therefore, this number is much smaller than that of the coverings. We also give an illustrative example to show how we can classify some given coverings by calculating the topologies.

Moreover, based on the relationship between the neighborhood and the complementary neighborhood, we find that each class of coverings has a dual class, and each pair of approximation operators has a dual pair of approximation operators. For a finite universe, considering one pair of approximation operators under one equivalence class of coverings is equivalent to considering its dual approximation operator pair under the dual class of coverings. Finally, we also present a sufficient condition under which, for any given pair of neighborhood-based lower and upper approximation operators, two coverings of the universe U generate the same pair of approximations of each $X \subset U$ if and only if they belong to the same equivalence class.

1. Introduction

Rough set theory as well as fuzzy set theory are generalizations of the classical set theory for modelling vagueness and uncertainty in the study of intelligent systems which are characterized by insufficient and incomplete information [4,10,17,18]. The covering rough set theory, which was established by Zakowski [30] in 1983, is a generalization of Pawlak's original equivalence-relation-based rough set theory [14–16]. As it can handle more complex practical problems, this theory has drawn the interest and attention of researchers in various fields of sciences [1,2,7,20,24,28,30,31,33,34,36]. Recently, the concept of neighborhood has been introduced to define and study different types of neighborhood-based covering rough sets [8,9,11,32,35]. The neighborhood-based covering rough sets have been demonstrated to be useful in the discovering of decision rules from the incomplete information systems [29].

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http://dx.doi.org/10.1016/j.ins.2014.02.045
0020-0255/

There are many ways to define the lower and upper approximations for subsets of a universe. If, besides the neighborhoods (or complementary neighborhoods) of the elements, the definition of a pair of lower and upper approximations $l^-(X),\ l^+(X)$ (or $L^-(X),\ L^+(X)$) of $X \subset U$ involves only the elements of U, subsets of U and logic operation symbols, we say that this type of lower and upper approximation pair is neighborhood (or complementary neighborhood)-based lower and upper approximation pair, or we say that l^- and l^+ (or L^- and L^+) are neighborhood (or complementary neighborhood)-based lower and upper approximation operators on U. For example, the lower and upper approximation operators in Definition 10 are all neighborhood-based lower and upper approximation operators. For a universe U with n points, the number of possible different coverings of U can be very large when $n > 3$, and we can calculate it by the formula $\frac{1}{2}\sum_{k=0}^{n}(-1)^k\binom{n}{k}2^{2^{n-k}}$ [25]. Since different coverings of U may generate the same neighborhood of each $x \in U$, for a given pair of neighborhood-based lower and upper approximation operators, different coverings may lead to the same lower and upper approximations of each $X \subset U$. Therefore, it would be of great practical and theoretical significance to consider the problem of covering classification in the covering rough set theory. This problem contains two aspects: Firstly, classifying all the coverings of U so that for arbitrary pair of neighborhood-based lower and upper approximation operators, the coverings in the same class generate the same pair of lower and upper approximations of each $X \subset U$. Secondly, evaluating the number of equivalence classes of coverings.

In this paper, the covering classification problem is considered based on the notions of neighborhood and complementary neighborhood. We present a classification rule under which the coverings are classified such that for any given pair of neighborhood (or complementary neighborhood)-based lower and upper approximation operators, all the different coverings in the same class generate the same pair of lower and upper approximations of any given subset of the universe.

Topology is an effective tool to study the rough set theory and other subjects [11–13,19,21,26,27,35]. By investigating the connections between coverings and topologies of the universe, we reveal that the number of equivalence classes of coverings can be evaluated by counting the number of topologies of the universe. We also give a practical way to classify some given coverings of the universe.

Moreover, based on the properties of the topology and the connections between the neighborhood and complementary neighborhood, we find that each equivalence class of coverings has a dual class, and each pair of neighborhood-based approximation operators has a dual pair of complementary neighborhood-based approximation operators. Properties of the dual approximation operator pairs and the dual covering classes are also investigated.

An ideal covering classification rule is such that different coverings belonging to the same equivalence class if and only if for any given pair of lower and upper approximation operators, they generate the same pair of lower and upper approximations of each subset of the universe. However, we show through an example that our classification rule is not ideal, i.e. given a pair of neighborhood-based lower and upper approximation operators, two coverings of the universe may generate the same pair of approximations of each $X \subset U$ even if they belong to different equivalence classes. This makes us to consider the problem of under what condition our classification is ideal.

This paper is organized as follows. In Section 2, we review some basic notions and results in the covering rough set theory and topology. In the subsequent section, we present the covering classification rule and reveal the relationship between the topologies and the equivalence classes of coverings. The definitions of complementary neighborhood, dual covering classes and dual approximation operator pairs are given in Section 4, and the properties of them are investigated. In Section 5, we consider the connection between the approximation operators and the class of coverings. We conclude in the last section.

2. Basic notions in covering rough set theory and topology

In this section, we will recall some fundamental concepts and properties in rough set theory and topology.

Let U be a set called the universe, and R be an equivalence relation on U. Denote by U/R the family of all equivalence classes induced by R. Obviously U/R gives a partition of U. For any $X \subseteq U$, the lower and upper approximations of X are defined as

$$R_*(X) = \cup\{Y_i \in U/R : Y_i \subseteq X\}, \quad R^*(X) = \cup\{Y_i \in U/R : Y_i \cap X \neq \emptyset\}.$$

According to Pawlak's classical definition, X is called a rough set if and only if $R_*(X) \neq R^*(X)$.

By relaxing the equivalence classes to coverings, the covering rough set theory has been proposed and the concept of neighborhood has been used to define the lower and upper approximations of covering rough sets.

Definition 1 [35]. Let U be a universe and $\mathcal{C}$ be a family of subsets of U. If no element in $\mathcal{C}$ is empty and $U = \cup_{C\in\mathcal{C}}C$, then $\mathcal{C}$ is called a covering of U, and the ordered pair $(U, \mathcal{C})$ is called a covering approximation space.

In what follows, we denote $U - X$ by $-X$ for every subset X of U.

In the sense of Pawlak, a set $X \subset U$ is called a covering rough set if its covering-induced lower and upper approximations are not equal.

Definition 2 [35]. Let $(U, \mathcal{C})$ be a covering approximation space. We define the neighborhood of an element $x \in U$ as

$$N(x) = \cap\{C \in \mathcal{C} : x \in C\}.$$