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Why are papers about filters on residuated structures (usually) trivial? ☆

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ABSTRACT

In this paper we introduce a notion of a *t*-filter on residuated lattices which is a generalization of several special types of filters. We provide some basic properties of *t*-filters and show how particular results about special types of filters (e.g. Extension property, Triple of equivalent characteristics, and Quotient characteristics) are uniformly covered by this simple general framework.

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1. Introduction

Papers about special types of filters on residuated structures belong to a 'folklore' of journals concerning fuzzy logics, sets and systems. The main goal of this short paper is to provide an insight into the numerous existing results about special types of filters (implicative, fantastic, etc.) on several different algebras of non-classical logics, including BL-algebras, MTL-algebras and FL_{ew}-algebras.

We demonstrate that these results can be seen as straightforward consequences of simple general principles which are explained here assuming that an (usually very easy to prove) equivalent characterization of concrete special type of filter in question is provided.

We do not want to increase the amount of papers about particular, artificial types of filters. We want to illuminate the triviality of the 'theory' behind these papers. Proofs of presented general claims are short and clear – in contrast to proofs of particular results for concrete special types of filters which are technical and they seem like "math exercises". We also want to provide a tool for reviewers who battle with dozens of papers dealing with unmotivated types of filters.

Our aim is to cover the great amount of papers about special types of filters—e.g. (positive) implicative, boolean, fantastic, IMTL-filters—which were published in many journals. In order to illuminate these results we define a new notion of a *t*-filter which generalizes among others these special types of filters in a uniform way. After that we state and prove theorems that are 'patterns' of particular theorems ('Extension property', 'Triple equivalent characteristics' and 'Quotient characteristics') appearing often in the literature. Even though the proofs of those general theorems are very simple, our approach is vindicated by the sheer numbers of particular theorems in published papers it covers.

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Referenced papers contain—above all— results of three particular kinds, which we are going to illustrate in the case of BL-algebras (for the notion of a BL-algebra see [2]) and so-called fantastic filters.

2. Preliminaries

Definition 1 [8, Definition 4.1]. A nonempty subset F of a BL-algebra $\boldsymbol{A}$ called a fantastic filter if it satisfies:

1. $\bar{1} \in F$
2. $z \to (y \to x) \in F$ and $z \in F$ imply $((x \to y) \to y) \to x \in F$ for all $x, y, z \in A$.

Theorem 2 (*Extension property, [8, Theorem 4.1]*). *Suppose $F \subseteq G$ where F is a fantastic filter and G is a filter of (a BL-algebra) $\boldsymbol{A}$. Then G is a fantastic filter.*

Theorem 3 (*Triple of equivalent characteristics, [8, Theorem 4.9]*). *In a BL-algebra $\boldsymbol{A}$, the following conditions are equivalent:*

1. *$\{\bar{1}\}$ is a fantastic filter.*
2. *Every filter on $\boldsymbol{A}$ is a fantastic filter.*
3. *$\boldsymbol{A}$ is an MV-algebra.*

Theorem 4 (*Quotient characteristics, [8, Theorem 4.10]*). *Let F be a filter of (a BL-algebra) $\boldsymbol{A}$. Then F is a fantastic filter if and only if every filter of the Quotient algebra $\boldsymbol{A}/F$ is a fantastic filter.*

In this text we are going to focus on filters definable via a certain term t. But first, we are going to recall two necessary definitions.

Definition 5 [5]. A bounded pointed commutative residuated lattice, or an FL_e^b-algebra for short, is an algebra $\boldsymbol{L} = \langle L, \&, \to, \wedge, \vee, \bar{0}, \bar{1}, \bot, \top \rangle$ such that:

1. $\langle L, \wedge, \vee, \bot, \top \rangle$ is a bounded lattice,
2. $\langle L, \&, \bar{1} \rangle$ is a commutative monoid,
3. $\to$ is the residuum of $\&$, i.e., for each $x, y, z \in L$ holds: $x \,\&\, y \leqslant z$ iff $x \leqslant y \to z$.

An FL_{ew}-algebra is an FL_e^b-algebra where $\bar{0} = \bot$ and $\bar{1} = \top$.[1]

Definition 6. Let $\boldsymbol{L}$ be an FL_e^b-algebra and $x, y \in L$. Then a set $F \subseteq L$ is called a filter on $\boldsymbol{L}$ if the following conditions are fulfilled:

1. if $x, y \in F$, then $x \,\&\, y \in F$,
2. if $x, y \in F$, then $x \wedge y \in F$,
3. if $x \leqslant y$ and $x \in F$, then $y \in F$,
4. $\bar{1} \in F$.

It can be shown that first condition can be equivalently replaced by 'closeness under modus ponens': if $x \in F$ and $(x \to y) \in F$, then $y \in F$; and the second one by the so-called $\bar{1}$-adjunction: if $x \in F$, then $x \wedge \bar{1} \in F$. Note that the set $\{x \in L | \bar{1} \leqslant x\}$ is a filter on any FL_e^b-algebra $\boldsymbol{L}$ and it is the minimum filter with respect to inclusion, i.e. it is included in any filter. A connection with the notion of a logical filter (L-filter) is quite clear when a definition of an L-filter is considered.

Recall the definition: Let $\boldsymbol{A}$ be an $\mathcal{L}$-algebra. We say that $F \subseteq \boldsymbol{A}$ is an L-filter, if for each $\Gamma \cup \{\varphi\} \subseteq Fm_{\mathcal{L}} : \Gamma \vdash \varphi$ implies that for each $\boldsymbol{A}$-evaluation e, we have $e(\varphi) \in F$ whenever $e[\Gamma] \subseteq F$ ($\mathcal{L}$ is an assumed language). This connection is slightly described in [10] and comprehensively studied in [3].

3. General results on *t*-filters

In the further text we will use sometimes $\bar{x}$ as an abbreviation of a finite sequence $x_1, x_2, \ldots$.

Definition 7. Let $t(\bar{x})$ be a term of the language of FL_e^b-algebras. We say that a filter F on an FL_e^b-algebra $\boldsymbol{L}$ is a t-filter if $t(\bar{x}) \in F$ for all $\bar{x} \in L$.

[1] Note that in fuzzy logic literature, following [6], these algebras are called shortly, but from a modern perspective unsystematically, just *residuated lattices*. In the definitions of structures involved, we require the commutativity of $\&$, but in fact the analogous results can be stated also in case of non-commutative structures. The choice of commutative structures was done mainly due to clarity and the fact that all of results we generalize were proved for commutative structures.

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