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Approximate reasoning algorithm of interval-valued fuzzy sets based on least square method ☆

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ABSTRACT

In this paper, we develop a new method for bidirectional approximate reasoning of interval-valued fuzzy sets by employing least squares method and investigate some properties for our proposed approximate reasoning method. Furthermore, we use four numerical examples to illustrate our proposed method reasonable.

1. Introduction

Numerous algorithms for handling approximate (fuzzy) reasoning problems based on fuzzy set theory have been proposed by some researchers [1,3,4,7,12,16,17,22] since fuzzy set was introduced by Zadeh [22]. The following single-input–single-output (SISO) approximate reasoning scheme is widely discussed by many researchers.

$$R : \text{IF } x \text{ is } A \text{ THEN } y \text{ is } B$$
$$\text{Fact} : x \text{ is } A^*$$
$$\text{Consequence} : y \text{ is } B^*$$

where R is fuzzy production rule [17], x and y are linguistic variables [23], A, B, A^* and B^* are linguistics terms represented by fuzzy sets [22]. For example, Bien and Chun [3] presented an inference network for bidirectional approximate reasoning based on fuzzy sets. Zadeh [22] and Fukami et al. [12] investigated their conclusions on fuzzy inference, respectively. Mizumoto and Zimmermann [16] made some comparisons with some different fuzzy reasoning methods. Bustince et al. [4] studied the generalized modus ponens (GMP) and the generalized modus tollens (GMT) in a different way by merging the least square method into fuzzy inference. Zhang and Yang [30] investigated some properties of fuzzy reasoning in propositional fuzzy logic systems. These approximate reasoning methods have been applied in some different fields. For example,

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An et al. [2] investigated the fuzzy reasoning and fuzzy AHP and applied in the railway risk management. Chun [11] applied the similarity-based bidirectional approximate method in decision making system. Wang et al. [20] proposed a dynamic fuzzy inference marginal linearization method and applied in a class of non-autonomous systems.

Interval-valued fuzzy set is the generalization of ordinary fuzzy set introduced by Zadeh [23–25]. Since then, some scholars have investigated this topic and obtained some meaningful results. For example, Zeng et al. [26–29] investigated decomposition theorems, representation theorems and the relationship among normalized distance, similarity measure, inclusion measure and entropy of interval-valued fuzzy sets. Gorzalczany [13,14] presented a method for interval-valued fuzzy reasoning based on compatibility measure and studied some properties about the reasoning method, respectively. Turksen [19] investigated the interval-valued fuzzy set based on normal forms. Yuan et al. [21] investigated the normal form based on interval-valued fuzzy set and applied in approximate reasoning. Chen et al. [8,9] proposed two methods based on similarity measure and the direction index of matching function between interval-valued fuzzy sets and applied them to bidirectional approximate reasoning for rule-based systems. Pedrycz [18] investigated the process of decision making of interval-valued fuzzy set and applied it in intelligent system. Chen et al. [10] presented a weighted fuzzy interpolative reasoning method for sparse fuzzy rule-based systems based on the ranking values of interval type-2 fuzzy sets. Bustince [5,6] investigated interval-valued fuzzy relation and indicator of inclusion grade of interval-valued fuzzy sets, and applied them in approximate reasoning of interval-valued fuzzy sets, respectively. Li et al. [15] investigated the robustness of interval-valued fuzzy reasoning in terms of the sensitivity of interval-valued fuzzy connectives and maximum perturbation of interval-valued fuzzy sets. However, these algorithms mainly concentrated on the information measure of interval-valued fuzzy sets such as similarity measure and compatibility measure, and so on.

Consider the following generalized modus ponens (GMP) and generalized modus tollens (GMT):

$$\begin{aligned}&\text{Rule}: \text{IF } x \text{ is } A \text{ THEN } y \text{ is } B\\&\text{Fact}: x \text{ is } A^*\\&\text{Consequence}: y \text{ is } B^*\end{aligned}$$

$$\begin{aligned}&\text{Rule}: \text{IF } x \text{ is } A \text{ THEN } y \text{ is } B\\&\text{Fact}: y \text{ is } B^*\\&\text{Consequence}: x \text{ is } A^*\end{aligned}$$

where x and y are linguistic variables; A^* and A are interval-valued fuzzy sets of the discourse set $U\,(U=\{u_1,u_2,\ldots,u_n\})$; B^* and B are interval-valued fuzzy sets of the discourse set $V\,(V=\{v_1,v_2,\ldots,v_m\})$.

Chen et al. [9] proposed the bidirectional approximate reasoning of interval-valued fuzzy sets using the direction index of matching function which can satisfy the following properties.

(a) if $A^*=$ very A, then $B^*=$ very B;
(b) if $A^*=$ more or less A, then $B^*=$ more or less B;
(c) if $B^*=$ very B, then $A^*=$ very A;
(d) if $B^*=$ more or less B, then $A^*=$ more or less A.

In this paper, we are inspired by the work of Bustince [4] and develop a novel bidirectional approximate reasoning method of interval-valued fuzzy sets based on least squares method. Our proposed method not only satisfies the properties (a–d), but also has a good validity in the general cases. It can provide a more reasonable and powerful way to deal with bidirectional approximate reasoning for rule-based systems.

The rest of our work is organized as follows. In Section 2, we briefly recall some notions of interval-valued fuzzy sets. In Section 3, we propose a novel method for bidirectional approximate reasoning of interval-valued fuzzy sets based on least squares method in detail, and prove two main theoretical results. In Section 4, we use four numerical examples to illustrate our proposed method reasonable. The conclusion is given in the last section.

2. Preliminaries

Throughout this paper, let $U=\{u_1,u_2,\ldots,u_n\}$ and $V=\{v_1,v_2,\ldots,v_m\}$ denote the discourse sets of input and output, respectively. IVFSs(U) and IVFSs(V) stand for the set of all interval-valued fuzzy sets of the discourse sets U and V, respectively. $\mathbb{R}$ stands for the set of all real numbers. $L([0,1])$ stands for the set of all closed subintervals of the interval $[0,1]$.

We call a mapping $A: U \to L([0,1])$ an interval-valued fuzzy set of the discourse set U. For every $A \in$ IVFSs(U) and $u_i \in U$, then $A(u_i)=[A^-(u_i),A^+(u_i)]=[a_{i1},a_{i2}]\,(0 \leqslant a_{i1} \leqslant a_{i2} \leqslant 1, i=1,2,\ldots,n)$ is the degree of membership of an element u_i to the interval-valued fuzzy set A. Then we have

$$A=\{(u_1,[a_{11},a_{12}]),(u_2,[a_{21},a_{22}]),\ldots,(u_n,[a_{n1},a_{n2}])\}$$

For $A, A_1, A_2 \in$ IVFSs(U), the following operations such as union, intersection and complement can be found in Zeng et al. [26].