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On reliability of the folded hypercubes in terms of the extra edge-connectivity

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A B S T R A C T

For a graph G and a non-negative integer g, the g-extra edge connectivity of G is the minimum cardinality of a set of edges in G, if it exists, whose deletion disconnects G and each remaining component will have at least g vertices. The extra edge-connectivity is an important parameters for the reliability evaluation of interconnection networks. In this paper, we explore g-extra-edge-connectivity ($\lambda_g(FQ_n)$) of the folded hypercube FQ_n for $g \leqslant n$ (denote g by $\sum_{i=0}^{s} 2^{t_i}$, where $t_0 = [\log_2 g]$ and $t_i = \left[\log_2\left(g - \sum_{r=0}^{i-1} 2^{t_r}\right)\right]$). We show that $\lambda_g(FQ_n) = g(n+1) - \left(\sum_{i=0}^{s} t_i 2^{t_i} + \sum_{i=0}^{s} 2 \cdot i \cdot 2^{t_i}\right)$ for $n \geqslant 6$. This result generalizes the previous results by Zhu et al. (2007) for $\lambda_3(FQ_n)$, and by Hsieh and Tsai (in press) for $\lambda_4(FQ_n)$, and so on.

1. Introduction

In a network, traditional edge-connectivity is an important measure since it can correctly reflect the fault tolerance of network systems with few processors. However, it always underestimates the resilience of large networks. There is a discrepancy because the occurrence of events which would disrupt a large network after a few link failures is highly unlikely. Thus the disruption envisaged occurs in a worst case scenario. To overcome this shortcoming, Esfahanian in [4] introduced the restricted edge-connectivity.

Since in some large interconnection networks, like hypercubes and star graphs, the degree of each vertex is large, there is a very small possibility that all the incident edges of a vertex fail simultaneously as we mentioned above. So it is safe to assume that all the incident edges of any vertex will not fail at the same time (this is the restriction). Thus, the restricted edge-connectivity is a more accurate measure of reliability of these interconnection networks. The restricted edge-connectivity of many interconnection networks have been shown to be about twice their traditional edge-connectivity.

One may even go further and ask what happens when more, even linearly many (exponential many) edges are deleted. To explore this problem, the extra edge-connectivity was introduced by several authors in, for example, [4,5,7], which generalizes the restricted edge-connectivity of graphs. In particular, Harary introduced the concept of conditional edge-connectivity [7] containing the extra edge-connectivity as a special case. The extra edge-connectivity of various classes of graphs were examined recently, see [2–5,8,13,14,16,18,19,22–25] and the references therein.

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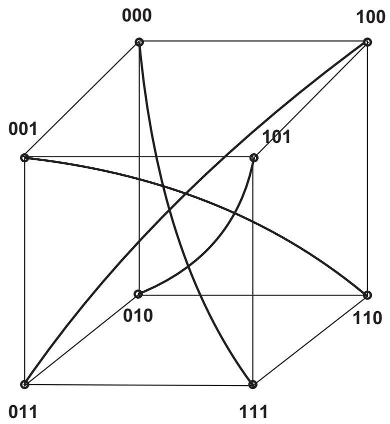

Fig. 1. The 3-dimensional folded hypercube.

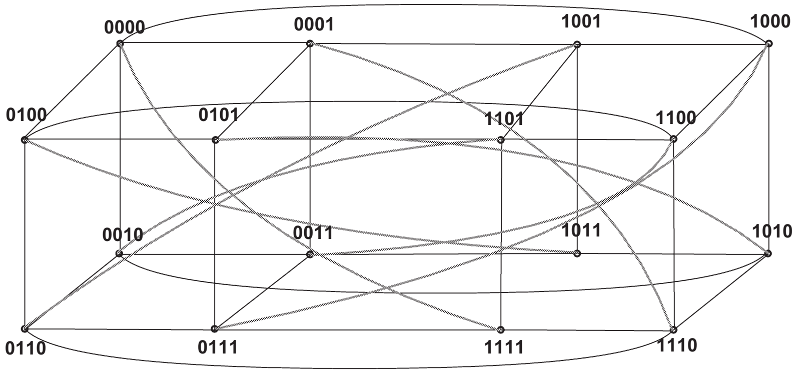

Fig. 2. The 4-dimensional folded hypercube.

For a connected graph G, an edge set F is called a *edge-cut* of G if $G - F$ is disconnected. A *g-extra edge-cut* of G is an edge-cut F of G such that every component of $G - F$ has at least g vertices. The cardinality of the minimum g-extra edge-cut of G is the *g-extra edge-connectivity* of G, denoted by $\lambda_g(G)$. A minimum g-extra edge-cut of G is abbreviated as a λ_g-cut of G. The extra edge-connectivity of folded hypercubes and several well-known cube-based graphs have attracted much attention these years, see for example [4,8,11,14,16,17,20–24]. However, the results are all around the special cases for small $g(\leq 4)$. In this note, we shall explore the g-extra edge-connectivity of folded hypercubes for more general $g \leqslant n$.

The *hypercube* is one of the most famous interconnected network models. An n-dimensional hypercube is an undirected graph $Q_n = (V, E)$ with $|V| = 2^n$ and $|E| = n2^{n-1}$. Each vertex can be represented by an n-bit binary string. There is an edge between two vertices whenever their binary string representation differs in only one bit position. Following Latifi et al. [14], we express Q_n as $D_0 \odot D_1$, where D_0 and D_1 are the two $(n-1)$-dimensional subcube of Q_n induced by the vertices with the *ith* coordinate 0 and 1 respectively. Sometimes we use $X_1 \ldots X_{i-1}0X_{i+1} \ldots X_n$ to denote a $(n-1)$-dimensional subcube of Q_n, where $X \in Z_2$, Moreover, if we fix $n-k$ coordinates of the n-bit binary string, or simply, we use $X_1X_2 \ldots X_k$ to denote a k-dimensional subcube in Q_n. Clearly, the vertex v in one $(n-1)$-subcube has exactly one neighbor v' in the other $(n-1)$-subcube. Let $u = x_1x_2 \ldots x_i \ldots x_n \in V(Q_n)$, we use $\bar{u} = \bar{x_1}\bar{x_2} \ldots \bar{x_n}$ to denote a vertex such that all coordinates of $\bar{u}$ are different from u's, i.e., $\bar{x_i}$ represents the complement of x_i. We use P_k to denote a path with k vertices withe length $k-1, d_G(u, v)$ to denote the distance of u, v in G. It is well known that $d_{Q_n}(u, \bar{u}) = n$.

The folded hypercube FQ_n is obtained by adding a prefect matching M on the hypercube, where $M = \{(u, \bar{u})|\ u \in V(Q_n)\}$. The 3-dimensional and 4-dimensional folded hypercubes are shown in the following Figs. 1 and 2 respectively. In particular, FQ_n is superior to Q_n in some properties, see [1]. Thus, the folded hypercube FQ_n is an enhancement on the hypercube Q_n. We suggest the readers to refer to [9,10,12,15] (and the reference therein) for the details on the properties of FQ_n. In addition, we denote by M_i the edge set $\{(x_1x_2 \ldots x_{i-1}x_ix_{i+1} \ldots x_n, x_1x_2 \ldots x_{i-1}\bar{x}_ix_{i+1} \ldots x_n) : x_i \in Z_2\}$. Clearly, $E(Q_n) = \cup_{i=1}^{n} M_i$ and $E(FQ_n) = E(Q_n) \cup M$, see [6].

Several special cases of the g-extra edge-connectivity of folded hypercubes have been reported in [1,11,20,22,23], in which the authors showed $\lambda_1(FQ_n) = n + 1, \lambda_2(FQ_n) = 2n$ for $n \geqslant 3$, $\lambda_3(FQ_n) = 3n - 1$ for $n \geqslant 4$, and $\lambda_4(FQ_n) = 4n - 4$ for $n \geqslant 4$. However, these results all focus on several special cases of $\lambda_g(FQ_n)$, i.e., there is no general results on $\lambda_g(FQ_n)$ in