Information Sciences 259 (2014) 16–24

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Gradually tolerant constraint method for fuzzy portfolio based on possibility theory

Xue Deng [a,*], Rongjun Li [b]

[a] Department of Mathematics, School of Science, South China University of Technology, Guangzhou 510640, China
[b] School of Business Administration, South China University of Technology, Guangzhou 510640, China

ARTICLE INFO

Article history:
Received 7 November 2010
Received in revised form 13 September 2013
Accepted 13 October 2013
Available online 19 October 2013

Keywords:
Fuzzy number
Gradually tolerant constraint method
Portfolio selection
Possibilistic mean
Possibilistic variance

ABSTRACT

In financial markets, some nonprobabilistic factors can be modeled as fuzzy numbers. Based on possibility theory and the assumption that the returns of assets are triangular fuzzy numbers, a bi-objective nonlinear portfolio selection model is proposed in this paper. This model aims to maximize the future expected return and minimize the future expected risk. Moreover, the obtained nonlinear bi-objective model is equivalent to the linear bi-objective minimizing programming model on the basis of possibilistic mean and possibilistic variance. Using the gradually tolerant constraint method proposed in this paper, we give a numerical example to illustrate the efficiency of the proposed model and method. The proposed method in this paper has improvements in two aspects. One is that our method offers several satisfactory solutions for the same model compared with the linear weighted method of Chang (2009), which offers only one satisfactory solution according to the investor's risk preference degree. The other is that the effective frontier of our method is more efficient than that of the method proposed by Markowitz (1987) and Zhang (2007).

1. Introduction

Portfolio selection is concerned with the allocation problem of one's wealth among alternative securities in order to achieve a particular investment goal. The well-known probabilistic mean–variance model originally introduced by Markowitz [22,23] plays an important role in the development of modern portfolio selection theory. It combines probability theory with optimization techniques to model the behavior investment with some uncertainties. The key principle of the mean–variance model is to use the expected mean of the return as the measurement of investment return and the variance of the return as the measurement of investment risk. As we know, Markowitz's portfolio model is a bi-criteria optimization program, which either minimizes the risk for a given level of return or maximizes the return for a given level of risk, and achieves a reasonable trade-off between return and risk. Many researchers, such as Fang [15], Giove [16], Merton [25] and Sharpe [26], did a great deal of research work on portfolio selection. Specifically, Xue et al. [27] studied the mean–variance portfolio model with concave transaction cost; Chang et al. [10] constructed an enhanced process based on the investment satisfactory capability index and ensured the investment risk control by using particle swarm optimization algorithm; Dastkhan et al. [12] proposed the linguistic-based portfolio selection model using hybrid genetic algorithm with weighted max–min operator, and a practical example was verified the effectiveness of the algorithm.

* Corresponding author. Tel.: +86 13711194586.
E-mail address: dxue@scut.edu.cn (X. Deng).

0020-0255/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.ins.2013.10.016

In the past, a number of researchers investigated the fuzzy portfolio selection problem. Bellman and Zadeh [7] proposed the basic fuzzy decision theory. Carlsson and Fullér [9] discussed some basic properties about possibilistic mean and possibilistic variance of fuzzy numbers when the variable had some fuzzy uncertainties. Moreover, Amelia et al. [1] presented a model to select portfolios when an ethical dimension on financial products is considered. They combined a mathematical programming technique with multiple criteria: goal programming models, flexible targets and constraints. In the portfolio model, Arenas et al. [5] took three criteria into account: return, risk and liquidity. Considering the uncertain returns of risky assets as fuzzy numbers, Deng [13] obtained some new results on value ranges of risks for mean–variance portfolio models; Deng [14] also proposed a portfolio selection model with borrowing constraint based on possibility theory; Ida [19] considered the portfolio selection problem with interval and fuzzy objective function coefficients as a kind of multi-objective problem; Li et al. [21] presented a mean–variance-skewness model in 2010, which extended the fuzzy mean–variance model; Huang [18] presented a new perspective for the portfolio selection model; Zhang et al. [28] obtained the effective frontiers for portfolio possibilistic mean–variance models.

Furthermore, some researchers also paid more attention to the multi-objective portfolio selection models. These studies mainly focused on the methods of building up model, measuring risk and developing appropriate algorithm. As a result, the following models were proposed: the multi-objective nadir compromise portfolio programming model by Amiri [2], the fuzzy tri-objective mean–variance-skewness model with interval analysis by Bhattacharyya [6], the multi-objective credibilistic model with fuzzy chance-constraints by Gupta [17] and the multi-objective model with a compromise approach-based genetic algorithm by Li [20]. In order to find out the tradeoffs among risk, return and the number of securities, a tri-objective optimization portfolio selection model is presented by Anagnostopoulos et al. [3,4]. A multi-objective binary programming model is proposed by Carazo et al. [8] in order to arrange the optimum time with no need for a priori information of the decision-maker's preferences.

The proposed model in this work considers many objectives without requiring a priori specification of decision-maker's preferences. Considering the returns of risky assets as triangular fuzzy numbers, a bi-objective nonlinear portfolio selection model is proposed by maximizing the future expected return and minimizing the future risk. Furthermore, the obtained nonlinear bi-objective model can be transformed into a linear bi-objective minimizing programming model. By using gradually tolerant constraint method, we present a numerical example of the portfolio selection problem to illustrate the proposed effective possibilistic mean and effective possibilistic variance.

The rest of this paper is organized as follows. In Section 2, the possibilistic mean, possibilistic variance and possibilistic covariance are given. In Section 3, a bi-objective possibilistic efficient portfolio selection model based on possibility theory is proposed. In Section 4, the basic ideas and the computation steps of gradually tolerant constraint method are given. In Section 5, a numerical example is presented to verify the efficiency of the proposed model and method. The conclusion and discussion of the improvements of gradually tolerant constraint method are presented in Section 6.

2. Preliminaries

In this section, some basic definitions are introduced. A fuzzy number A is a fuzzy set of the real line $\mathbf{R}$ with a normal, fuzzy convex and continuous membership function of bounded support. The family of fuzzy numbers is denoted by $\mathbf{F}$. A γ-level set of a fuzzy number A is defined by $[A]^\gamma = \{t \in \mathbf{R} | A(t) \geqslant \gamma\}$ if $\gamma > 0$ and $[A]^\gamma = \mathrm{cl}\{t \in \mathbf{R} | A(t) > 0\}$ (the closure of the support of A) if $\gamma = 0$. Let A and B be fuzzy numbers with $[A]^\gamma = [a_1(\gamma), a_2(\gamma)]$ and $[B]^\gamma = [b_1(\gamma), b_2(\gamma)]$, for $\gamma \in [0, 1]$.

Next, we recall some notions introduced by Carlsson and Fullér [9]. Define the possibilistic mean value of A by

$$M(A) = \int_0^1 \gamma(a_1(\gamma) + a_2(\gamma))\mathrm{d}\gamma, \tag{1}$$

denote the possibilistic variance of A as

$$Var(A) = \frac{1}{2}\int_0^1 \gamma(a_2(\gamma) - a_1(\gamma))^2\mathrm{d}\gamma, \tag{2}$$

and define the possibilistic covariance of A and B by

$$Cov(A,B) = \frac{1}{2}\int_0^1 \gamma(a_2(\gamma) - a_1(\gamma))(b_2(\gamma) - b_1(\gamma))\mathrm{d}\gamma. \tag{3}$$

The standard deviation of A is defined as $\sigma_A = \sqrt{Var(A)}$. Moreover, the possibilistic mean value and variance of linear combinations of fuzzy numbers can be easily computed by Carlsson and Fullér [9] in a similar manner as in probability theory (see Lemmas 2.1 and 2.2).

Lemma 2.1. *Let A and B be two fuzzy numbers and let λ, $\mu \in \mathbf{R}$, then*

$$Var(\lambda A + \mu B) = \lambda^2 Var(A) + \mu^2 Var(B) + 2|\lambda\mu| Cov(A,B). \tag{4}$$