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Intuitionistic fuzzy geometric interaction averaging operators and their application to multi-criteria decision making

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ABSTRACT

This paper proposes some new geometric operations on intuitionistic fuzzy sets (*IFSs*) based on probability non-membership (*PN*) function operator, probability membership (*PM*) function operator and probability hetergeneous (*PH*) operator, which are constructed from the probability point of view. The geometric interpretations of these operations are given. Moreover, we develop some intuitionistic fuzzy geometric interaction averaging (*IFGIA*) operators. The properties of these aggregation operators are investigated. The key advantage of the *IFGIA* operators is that the interactions between non-membership function and membership function of different *IFSs* are considered. Finally, an approach to multiple attributes decision making is given based on the proposed aggregation operators under intuitionistic fuzzy environment, and an example is illustrated to show the validity and feasibility of the proposed approach.

1. Introduction

Intuitionistic fuzzy sets (*IFSs*), as a generalization of fuzzy sets [42], developed by Atanassov [3], is a powerful tool to deal with vagueness.

Since information aggregation is a pervasive activity in daily life, many researches had been done on this issue [7,15,24,27,32,37,34,35,30,39,38,18,26]. Among them, the weighted geometric averaging (*WGA*) operator [24] and the ordered weighted geometric averaging (*OWGA*) operator [34] are the most common operators. On a basis of the multiplication operation by Atanassov [4] and power operation by De and Biswas [12] on intuitionistic fuzzy sets, Xu and Yager [37] proposed some intutionistic fuzzy geometric aggregation operators and applied them to multi-attribute decision making problems. After these pioneering works, more attentions have been paid to intuitionistic fuzzy multi-criteria decision making problems [1,2,5,6,8,10,11,13,14,19–23,25,29,33,36,41,43–51]. Dymova and Sevastjinov [13] presented a method to deal with the intuitionistic fuzzy multi-criteria decision making problems based on Dempster–Shefer theory of evidence. Ye [40] used entropy weight to get criteria weights and rank alternatives according to the correlation coefficients. Xu [33] developed intuitionistic fuzzy power aggregation operators. Xu and Xia [36] presented the induced generalized aggregation operators under interval-valued intuitionistic fuzzy environments. Zhu et al. [51] proposed hesitant fuzzy geometric Bonferroni means. Li et al. [19] investigated the relationship between the similarity measure and the entropy of *IFS*s. Zhang et al. [45] presented

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the intuitionistic fuzzy rough approximation operators and discussed their connections with special intuitionistic fuzzy relations.

However, it is found that the operational laws and geometric aggregation operators on intuitionistic fuzzy sets in [4,37] are not suitable to be used in the special circumstances. For example, suppose that A and B are two intuitionistic fuzzy sets, $A = \langle u_A, v_A\rangle$ and $u_A = 0$, $B = \langle u_B, v_B\rangle$ and $u_B \neq 0$, then according to the multiplication operation by Atanassov [4], we have $u_{A\otimes B} = 0$. Obviously, u_B is not accounted for at all, which is an undesirable feature of an averaging operation. Furthermore, the *IFHGA* operator [37] has similar problems. For example, if $A_i = \langle u_{A_i}, v_{A_i}\rangle, i = 1, 2, \ldots, n, i \neq k$ are a collection of intutionitisc fuzzy sets, $A_k = \langle 0, v_{A_k}\rangle$, and $v_{A_k} \neq 0$, then we have $u_{IFHGA_{\omega,w}(A_1,\ldots,A_n)} = 0$ by using aggregation law in [37]. It is obvious that $u_{A_i}(i = 1, 2, \ldots, n, i \neq k)$ have no effects on the aggregation result.

Motivated by the works of [4,37] and the idea of interactions between non-membership function and membership function of different intuitionistic fuzzy sets, we focus on developing some new geometric operations on intuitionistic fuzzy sets (*IFSs*) and giving the geometric interpretations of these operations. Based on the new operations, we propose some intuitionistic fuzzy geometric interaction aggregation operators, including the *IFWGIA* operator, the *IFOWGIA* operator and the *IFHGIA* operator, which are more practical for an averaging operator. By the comparison with the existing method, it is concluded that the method proposed in this paper is a good complement to the existing works on *IFS*s, especially when one of the membership degrees of intutionitisc fuzzy sets is zero.

The rest of the paper is organized as follows. Section 2 reviews some basic concepts. Section 3 introduces new geometric operations on intuitionistic fuzzy sets and gives the geometric interpretations of these operations. In Section 4, we develop the intuitionistic fuzzy geometric interaction averaging (*IFGIA*) operators, and investigate their properties. In Section 5, an approach to intuitionistic fuzzy multi-criteria decision making is given based on the proposed *IFHGIA* operator. In Section 6, a numerical example is illustrated to show the feasibility and validity of the new approach, and the comparison between the work of this paper and other corresponding works is presented systematically. Finally, Section 7 concludes the paper.

2. Preliminaries

The concept of fuzzy sets (*FSs*) was introduced by Zadeh [42]. Let X be a universe of discourse in the following.

Definition 1 [42]. A fuzzy set F in X is defined as follows: $F = \{\langle x, u_F(x)\rangle | x \in X\}$, where $u_F: X \to [0,1]$ is the membership function of the fuzzy set F, and $0 \leqslant u_F(x) \leqslant 1$.

Atanassov [3] generalized the fuzzy set to intuitionistic fuzzy set (IFS) by adding an hesitation degree.

Definition 2 [3]. An intuitionistic fuzzy set in X is an expression: $A = \{\langle x, u_A(x), v_A(x)\rangle | x \in X\}$, where the functions $u_A: X \to [0,1]$ and $v_A: X \to [0,1]$ define the degree of membership and the degree of nonmembership of the element $x \in X$ to A, and for every $x \in X$, $0 \leqslant u_A(x) + v_A(x) \leqslant 1$.

For each IFS A in X, if $\pi_A(x) = 1 - u_A(x) - v_A(x)$, for all $x \in X$, then $\pi_A(x)$ is called the degree of indeterminacy of the element x to the set A.

In practice, intuitionistic fuzzy numbers can be denoted as $A = \langle u, v\rangle$ [32,37].

For convenience, the sets of all the intuitionistic fuzzy numbers are denoted by *IFNs*.

Some basic operations, such as multiplication operation [4] and power operation [12], were introduced under intuitionistic fuzzy environment.

Definition 3 (*4,12*). Suppose that $A = \langle u_A, v_A\rangle$ and $B = \langle u_B, v_B\rangle$ are two intuitionistic fuzzy sets, then

$$(1)\ A \otimes B = \langle u_A \cdot u_B, v_A + v_B - v_A \cdot v_B\rangle \tag{1}$$

$$(2)\ A^\lambda = \left\langle u_A^\lambda, 1 - (1 - v_A)^\lambda \right\rangle, \lambda > 0 \tag{2}$$

Chen and Tan [9] proposed a score function $S(A) = u_A - v_A$ to evaluate the degree of suitability that an alternative satisfies a decision make's requirement under intuitionistic fuzzy environment, where A is an intuitionistic fuzzy set, and $A = \langle u_A, v_A\rangle$. The score of A is directly related to the deviation between u_A and v_A, i.e., the bigger the score of intuitionistic fuzzy set A, the larger the intuitionistic fuzzy set A.

Hong and Choi [16] presented an accuracy function $H(A) = u_A + v_A$ to evaluate the accuracy degree of the intuitionistic fuzzy set $A = \langle u_A, v_A\rangle$, where $0 \leqslant H(A) \leqslant 1$. The larger the value of $H(A)$, the higher the accuracy degree of intuitionistic fuzzy set A [32].

Based on score function [9] and accuracy function [16], Xu [32,37] gave the comparison law for intuitionistic fuzzy sets as follows.

Definition 4 (*32,37*). Let $A = \langle u_A, v_A\rangle$ and $B = \langle u_B, v_B\rangle$ be two intuitionistic fuzzy sets. Then $A < B$ if and only if
(i) $S(A) < S(B)$.
or
(ii) $S(A) = S(B)$ and $H(A) < H(B)$.