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Bisimulations for fuzzy transition systems revisited

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ABSTRACT

Bisimulation is a well-known behavioral equivalence for discrete event systems, and has recently been adopted and developed in fuzzy systems. In this paper, we propose a new bisimulation, i.e., the *group-by-group fuzzy bisimulation*, for fuzzy transition systems. It relaxes the fully matching requirement of the bisimulation definition proposed by Cao et al. (2010) [2], and can equate more pairs of states which are deemed to be equivalent intuitively, but which cannot be equated in previous definitions. We carry out a systematic investigation on this new notion of bisimulation. In particular, a fixed point characterization of the group-by-group fuzzy bisimilarity is given, based on which, we provide a polynomial-time algorithm to check whether two states in a fuzzy transition system are group-by-group fuzzy bisimilar. Moreover, a modal logic, which is an extension of the Hennessy–Milner logic, is presented to completely characterize the group-by-group fuzzy bisimilarity.

1. Introduction

Bisimulations are well-established forms of behavioral equivalences for discrete event systems, and have become a central notion in, for instance, process algebras, automata theory, etc. They are widely used in many areas of computer science, in particular, in verification where they are crucial to reduce the state space of the system under consideration.

Recently, bisimulations have been developed in fuzzy systems as well. For example, Cao et al. [2,4] considered bisimulations for *fuzzy transition systems* (FTS) where both fuzzy transitions and nondeterministic transitions co-exist. This model is further studied under fuzzy automata by Cao et al. [3] and Pan et al. [15]. Ćirić et al. [5] investigated bisimulations for fuzzy automata. Qiu and Deng [9], and Xing et al. [19] studied (bi)simulations for fuzzy discrete event systems. Fan [10] discussed fuzzy bisimulations for Gödel logic. Wu et al. [16–18] investigated algorithms and logical characterizations of bisimulations for FTS. For more information about fuzzy (bi)simulations, we also refer the readers to [6,7,11]. In addition, model checking for fuzzy systems was also studied [12–14].

A central question regarding any notion of bisimulation (or in general, any equivalence) is its distinguishing power. Namely, to which extent it will distinguish a pair of states. As a simple example, we presented two FTS in Fig. 1. Assuming that states s_i and t_i are equal for $i = 4, 5, 6$ (i.e., they cannot be distinguished in this case), and thus one can be easily convinced that s_i and t_i are also equal for $i = 1, 2, 3$, and s_i and t_j are *not* equal for $1 \le i \ne j \le 3$ since they have different enabled actions. We are mainly interested in whether states s and t can be related by the bisimulation under consideration.

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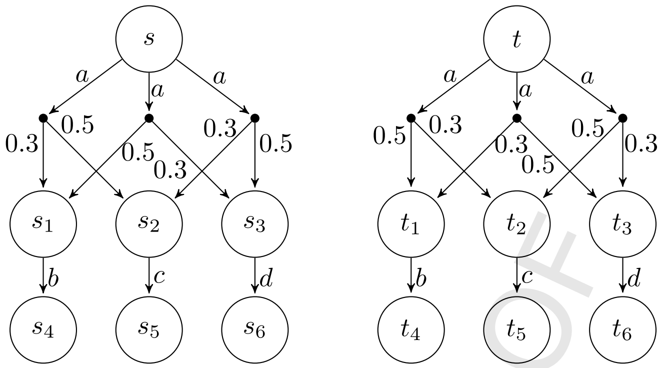

Fig. 1. s and t are group-by-group fuzzy bisimilar.

The bisimulation proposed by Cao et al. [4] will distinguish them. To see this, the transition $s \xrightarrow{a} \frac{0.3}{s_1} + \frac{0.5}{s_2}$ cannot be matched by either $t \xrightarrow{a} \frac{0.5}{t_1} + \frac{0.3}{t_2}$, or $t \xrightarrow{a} \frac{0.3}{t_1} + \frac{0.5}{t_3}$, or $t \xrightarrow{a} \frac{0.5}{t_2} + \frac{0.3}{t_3}$. However, arguably the two states should *not* be distinguished from the following perspective. The transition of $s \xrightarrow{a} \frac{0.3}{s_1} + \frac{0.5}{s_2}$ can be respectively matched by $t \xrightarrow{a} \frac{0.3}{t_1} + \frac{0.5}{t_3}$ (the central transition) when considering the *group of states* enabling only action b, $t \xrightarrow{a} \frac{0.5}{t_2} + \frac{0.3}{t_3}$ (the rightmost transition) when considering the *group of states* enabling only action c, and the $t \xrightarrow{a} \frac{0.5}{t_1} + \frac{0.3}{t_2}$ (the leftmost transition) when considering the *group of states* enabling action b or c. The other transitions from s can be analyzed similarly. From this point of view, s and t ought *not* be distinguished. Indeed, in [4] the bisimilar states must stepwise behave the same along two *fully matching resolutions*, which in this case unnecessary and should be relaxed.

The aim of this paper is to define a new bisimulation for an FTS, dubbed group-by-group fuzzy bisimulation. This bisimulation is different from the one proposed by Cao et al. [4] in two aspects: (1) The bisimulation in [4] considers each equivalence class of some equivalence relation R, while ours considers each subset of equivalence classes; this is why it is called a group-by-group fuzzy bisimulation, and (2) for a transition $s \xrightarrow{a} \mu$, the bisimulation in [4] requires that there exists a transition $t \xrightarrow{a} \nu$ such that μ and ν are equal at all equivalence classes of some equivalence relation R, i.e., the definition requires fully matching. In contrast, our definition requires that for each subset $\mathcal{G}$ of the equivalence classes and a transition $s \xrightarrow{a} \mu$, there exists a transition $t \xrightarrow{a} \nu$ such that μ and ν are equal for the union of $\mathcal{G}$. We note that for $\mathcal{G}_1$, ν_1 exists such that μ and ν_1 are equal for the union of $\mathcal{G}_1$, but for different $\mathcal{G}_2$, it is possible that ν_2 exists such that μ and ν_2 are equal for the union of $\mathcal{G}_2$. Loosely speaking, our new bisimulation allows *partially matching resolutions*.

We perform a systematic study on this new bisimulation by giving its characterization in different machinery, as follows.

1. We use a fixed point method to characterize group-by-group fuzzy bisimulation. This characterization shows that a group-by-group fuzzy bisimulation is a post-fixed point of some suitable monotonic function over a complete lattice, while a group-by-group fuzzy bisimilarity, the greatest group-by-group fuzzy bisimulation, is the greatest fixed point of this monotonic function.
2. We give a polynomial time algorithm to computing group-by-group fuzzy bisimilarity. Our algorithm follows the standard partition-refinement framework which is the cornerstone for the computation of almost all bisimulations in conventional labeled transition systems, and their various probabilistic and fuzzy extensions. In the current setting, while an exponential-time algorithm can be obtained almost for free, designing a polynomial-time algorithm turns out to be difficult simply because of the universal quantification over all subsets of the state space (cf. Definition 3). As a witness, for probabilistic systems considered in [1], a similar group-by-group probabilistic bisimulation is proposed, but eludes a polynomial-time algorithm.[1] In contrast, we show that in the fuzzy setting, a polynomial-time algorithm does exist, owing to that, essentially, the operations of max and min instead of addition and multiplication respectively are used.
3. We provide a logical characterization of a group-by-group fuzzy bisimilarity, which states that two states are group-by-group fuzzy bisimilar if and only if they satisfy the same logical formulae.

These characterizations suggest the robustness of our new definition of bisimulation of FTS. As mentioned, this work is inspired by the work in [1], where a group-by-group probabilistic bisimulation is investigated in probabilistic systems. However, it is probably noteworthy that our work is different from that in [1] in two aspects: (1) neither the fixed point characterization nor the algorithm was discussed in [1], and (2) [1] has given a logical characterization of group-by-group

[1] We conjecture such a polynomial-time algorithm does not exist.

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