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A generic framework to include belief functions in preference handling and multi-criteria decision ☆

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A B S T R A C T

Modelling the preferences of a decision maker about alternatives having multiple criteria usually starts by collecting preference information (comparisons of alternatives, importance of criteria, ...), which are then used to fit a preference model issued from some set of hypothesis (weighted average, CP-net, lexicographic orderings, AHP, ...). In practice, this process often leads to inconsistencies that may be due to inaccurate information provided by the decision maker, who can be unsure about the provided information, or to a poor choice of hypothesis set, which can be too restrictive or not well adapted to the decision process. In this paper, we propose to use belief functions as a way to quantify and resolve such inconsistencies, notably by allowing the decision maker to express her/his certainty about the provided preferential information. Our framework is generic, in the sense that it does not assume a given set of hypothesis a priori, and is consistent with precise methods, in the sense that in the absence of uncertainty and inconsistencies in the information, precise models are ultimately retrieved.

1. Introduction

Preference modelling and multi-criteria decision analysis (MCDA for short) are increasingly used in our everyday lives. Generally speaking, their goal is to help decision makers to model their preferences about multi-variate alternatives, to then formulate recommendations on unseen or unanalysed alternatives. Such recommendations can take various shapes, but three common problems can be differentiated [2]:

- **the choice problem**, in which a (set of) best alternative(s) has to be recommended to the decision maker (DM);
- **the ranking problem**, in which the alternatives have to be ranked before the ranking is presented to the DM;
- **the sorting problem**, in which each alternative has to be attributed to a class among a set of ordinal classes, the attribution being then presented to the decision maker.

In this paper, we will be interested in the first two problems, who are usually related, since the choice problem roughly consists in presenting only those elements that would be ranked highest in the ranking problem. The sorting problem can be considered to some extent as different, as it requires to first define the discrete classes, and only then can it be seen as the task of summarising the (complete) ordering of alternatives into these discrete classes. This latter problem is

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typically considered by outranking models [32,21], and is close to the learning problem known as ordinal regression or classification [12].

One common task, in preference modelling as well as in MCDA, is to collect or elicit preferences of decision makers (DM). This elicitation process can take various forms: from asking directly the values of the parameters to comparing known alternatives; from being static, where all preference information are collected at once, to being dynamic, where collected preference information depends on the process history; from aiming at identifying a preference model with enough precision, to aiming at making a robust recommendation of known alternatives (possibly with an imprecisely identified model). In all these cases, each piece of collected information then helps to better identify the preference model of the DM. Also, the elicitation processes may differ greatly between the different MCDA models, be it Choquet Integrals [28], GAI-networks [27], CP-net [6], lexicographic orderings [26,22] and their conditional extensions [4], ...

While we will not discuss in details how to perform the elicitation step in this paper, a common problem in this task is to ensure that the information provided by the DM is consistent with the chosen model, as the DM may be uncertain about the given preference information, or as the chosen model may not have a sufficient expressive power to describe accurately these preferences. It is therefore desirable to encode the uncertainty associated to the various pieces of information, as well as to combine them. Common ways to handle such a task is to identify the parameters of the preference models minimising some error term, for instance the quadratic error [28], to adopt a probabilistic version of preference models and to develop a corresponding probabilistic (e.g., Bayesian) learning method [48], or to relax the model constraints by following some minimal change principle [38]. While such methods try to solve inconsistency between the assessments and the chosen model in principled way, most do not consider the initial information to be uncertain.

Another problem when modelling preferences is to choose an adequate family of models, expressive enough to capture the DM preferences, but also sufficiently simple so that it can be identified with a reasonable amount of information. While there are some works around comparing the expressiveness of different model families [37,35], few of them have actually investigated how to choose a family among a set of possible ones.

In this paper, we propose to model uncertainty in preference information through belief functions, arguing that they can bring interesting answers to both issues (i.e., inconsistency handling and model choice). Indeed, belief functions are adequate models to model subjective uncertainty about non-statistical quantities (in our case the preferences of an individual decision maker), and many works have been devoted to the question of how to combine such information and assess the resulting inconsistency [18].

It is not the first work that tries to combine belief functions with MCDA and preference modelling, however the past works that dealt with such issues can be split into two main lines of works:

- those that start from a specific MCDA model and propose some adaptation to embed belief functions within it. This has mainly been done for the AHP model [3,24], but also for the TOPSIS model [20].
- those that start from belief functions defined on the various criteria, and then propose to perform inferences about preferences using such belief functions as well as tools issued from evidence theory, possibly but not necessarily inspired from existing MCDA techniques. This is for instance the case of the evidential reasoning approach [49] or the case of the outranking approach developed by Boujelben et al. [5].

The approach investigated and proposed in this paper differs from those works in at least two ways:

- we do not make any a priori assumption about the kind of model used, that is we do not start from an existing method and propose a corresponding extension. This means that the proposal can be directly applied to various methods (including those already well-studied);
- when selecting a particular model, we can retrieve the precise, certain version of the method as a particular instance of our approach, meaning that we are fully consistent with the standard models.

Section 2 describes the framework we propose, with an illustrative running example using weighted averages. Needed notions issued from evidence theory are introduced gradually when they are needed. To show that our approach can be adapted to various models, Section 3 provides three other illustrations to show the generality of our approach. The first one uses a simple dominance rules, the second a lexicographic qualitative approach to describe the provided information, and the last one is a simplified application of the popular AHP method. Section 4 then discusses how the framework of belief functions can be instrumental to deal with the problems we mentioned in this introduction: handling inconsistent assessments of the DM, and choosing a rich enough family of models. This paper is an extended version of [17], that beyond additional explanations, includes examples dealing with other models and an extended discussion of the solutions proposed for the ranking and choice problems.

2. The basic scheme

We assume that we want to describe preferences over alternatives X issued from a multivariate space $\mathscr{X} = \times_{i=1}^{C} \mathscr{X}^i$ constituted of C different criteria X^i. For instance, $\mathscr{X}$ may be the space of hotels, cars, applicants, meals ... and given criteria $\mathscr{X}^i$ may be the price, age, skill in a given language, main courses, ...