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Upper and lower probabilistic preferences in the graph model for conflict resolution

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ABSTRACT

We propose a model where decision makers may express their preferences among the possible conflict scenarios using upper and lower probabilities in the graph model for conflict resolution (GMCR). In this new model, we propose eight stability definitions (solution concepts) that are generalizations of the four stability concepts commonly used in the GMCR model, namely: cautious α-Nash stability, risky α-Nash stability, cautious (α, β)-metarationality, risky (α, β)-metarationality, cautious (α, β)-symmetric metarationality, risky (α, β)-symmetric metarationality, cautious (α, β, γ)-sequential stability and risky (α, β, γ)-sequential stability. We present these definitions for conflicts with two or more decision makers and also for conflicts in which the decision makers act as a coalition and analyze the relationship between them. We present two applications and perform the stability analysis using the proposed model to illustrate the gains obtained when individuals are allowed to have the uncertainty about their own preferences expressed by upper and lower probabilities.

1. Introduction

A *strategic conflict* is a situation involving two or more parties, who may, for example, be individuals, companies, countries or teams, and these parties, usually called decision makers (DMs), have to make choices [1]. The choices made by the DMs involved determine the conflict evolution and what are the possible scenarios that may arise, usually called states. Finally, each DM has preferences over the final conflict state, or resolution. Negotiation processes and conflicts are modeled and analyzed with perspectives coming from different fields, such as: operations research, computer science, psychology, political economy, systems engineering, social choice theory and game theory. Thus, there are conflict models used to determine how to design a reliable and efficient e-negotiation system [2] and also to improve our knowledge regarding the role of emotions in negotiations [3].

Fundamental aspects that have to be considered in the construction of a conflict model are to determine who are the DMs involved in the conflict, what are their available options and how to model their preferences over the possible conflict resolutions. The Graph Model for Conflict Resolution (GMCR) is a relevant and simple technique, based on some important game theoretical concepts [4], to represent conflicts. It was presented by Kilgour et al. [5], and is an enhancement of the conflict analysis of Fraser and Hipel [6] and metagames analysis [7]. In this model, considering the options available to

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those involved in a conflict, there is a set of states that can happen during its course. Individuals or institutions involved in a conflict, called DMs, have preferences over the set of possible states [8]. There is a collection of graphs in the GMCR, where each one of them represents the movements that a single DM can make from a state to another.

Once the conflict model that will be used is defined, the next step is to perform the stability analysis. This analysis is a fundamental and useful tool to better understand the possible conflict interactions. There are different stability definitions. In GMCR models, the usual stability notions are: Nash stability (R) [9,10], general metarationality (GMR) [7], symmetric metarationality (SMR) [7], sequential stability (SEQ) [6] and when there are more than two DMs, their coalitional counterparts are also analyzed [11]. According to each one of these definitions a state may be stable or not. In general, a state is stable if no DM has incentives to move away from it according to some criteria. DMs' preferences have a key role in the stability analysis, since their preferences over the states directly affect the incentives for moving from one state to another. However, in some cases, these preferences are imprecise. Some generalizations of the GMCR try to model preference characteristics that can better represent real world conflicts. In these works, different preference structures are used. For example, preference uncertainty for two-DM and multi-DM conflicts in Li et al. [12] and Li et al. [13], respectively, fuzzy preferences in Al-Mutairi et al. [14] and in Bashar et al. [15] and degree of preferences in Hamouda, Kilgour and Hipel [16]. Using probabilistic preferences is an alternative way to represent preference uncertainty. According to Campello de Souza [17], the use of probabilistic preferences can better model the fluctuations of behavior usually present in the choices of individuals. Rêgo and Santos [18] applied probabilistic preferences into the GMCR, allowing the DMs' preferences for one state a over another state b to be represented by a precise probability $P(a,b)$. From now on, this case will be called *precise* probabilistic preferences. Rêgo and Santos [18] argued that probabilistic preferences have an advantage over fuzzy preferences in the sense that it can be estimated from observed choices made by the DM and also that due to its interpretation it can be more easily elicited from experts.

However, in most real world conflicts information about one own preference may be vague or ambiguous and in such case requiring the existence of a unique probability distribution that models such preference is too demanding. The theory of imprecise probabilities generalizes standard probability theory to handle more accurately such situations [19]. Imprecise probability is a term used to encompass many models of uncertain quantification, such as: upper and lower probabilities [20, 21], belief functions [22], comparative probability orderings [23] and upper and lower previsions [19]. Beer et al. [24] provide an overview of applications of Imprecise Probability models in engineering analysis and Fetz and Oberguggenberger [25] propose methods to evaluate of upper and lower probabilities induced by functions of an imprecise random variable. Thus, there are two reasons that motivated us to extend the GMCR model with precise probabilistic preferences to use imprecise probabilistic preferences. The first one is that using imprecise probabilistic preferences imposes less burden in the elicitation process of DMs' preference since it requires less information. The second one is that this model is able to describe risky and cautious behavior which are not possible using precise probabilistic preferences.

In this work, we generalize the GMCR by modeling the DMs preferences using upper and lower probabilities [20]. The organization of the paper is as follows: in Section 2, a brief review of the GMCR and of the upper and lower probabilities literatures is made; in Sections 3 and 4, the GMCR with upper and lower probabilistic preferences is proposed and new stability definitions for the proposed model are given. First, we present the model for bilateral conflicts, showing results on the relationships between the proposed stability definitions. Then, we extend the model for conflicts with multiple DMs and we also define how to perform a coalitional analysis; in Section 5, two applications of the proposed model are presented illustrating its potentiality; and in Section 6 we finish with final comments. This work extends a preliminary work [26]. More specifically, it extends such model to represent conflicts with multiple DMs and also presents definitions to perform a coalitional analysis using the proposed model. Moreover, Theorem 2 formalizes the relationship between the proposed and the stability concepts of the GMCR with precise probabilistic preferences [18].

2. Review of literature

We start this section by reviewing the GMCR as defined by Kilgour et al. [5]. The GMCR consists of a set of graphs, one for each DM involved in the conflict, and for each one of these DMs a preference relation over the set of nodes or vertices of the graph. In each graph, the nodes represent the possible conflict resolutions, called states, and the arcs in the graph of DM i represent the possible state changes that DM i is able to make in the course of the conflict. Thus, all graphs in the GMCR have the same set of nodes and differ from each other only in what arcs are present. Formally, the set of DMs is denoted by $N = \{1, 2, 3, \ldots, n\}$ and $S = \{1, 2, \ldots, s\}$ is the set of possible states or scenarios of a conflict. A collection of directed graphs, $D_i = (S, A_i)$, $i \in N$, is used to model a conflict. Let $R_i(s)$ be the set of states to which DM i can move while at state s, i.e., $R_i(s) = \{t \in S : (s,t) \in A_i\}$. As in Rêgo and Santos [18], we assume that $s \in R_i(s)$, $\forall i \in N$ and $\forall s \in S$ since, in any given state, DMs can always choose to keep the *status quo* not switching states.

The GMCR also makes use of a set of asymmetric binary relations, denoted by $\succ_i$, $i \in N$, which are interpreted as follows: $x \succ_i y$, if object x is strictly preferred to object y for DM i. In general, such binary relation need not be transitive. We abuse notation using $x_1 \succ_i x_2 \succ_i \ldots x_{m-1} \succ_i x_m$ to represent DM i's ranking of the objects, meaning that DM i strictly prefers x_1 to each one of the other $m-1$ objects and that x_2 is strictly preferred by i to each one of the other $m-2$ objects which come after it in the sequence, and so on.

We aim to extend the GMCR by allowing DMs to have uncertainty about their own preferences and, moreover, such uncertainty can be vague or imprecise.