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Eigenfrequency and deflection analysis of layered structure using uncertain elastic properties – a fuzzy finite element approach

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ABSTRACT

This is the first-time a higher-order fuzzy finite element model has been proposed and implemented to compute the laminated structural responses (modal frequency and static deflection) numerically including the uncertain elastic properties. The composite structure is modelled via a higher-order mid-plane theory which accounts the parabolic variation of the shear stress in conjunction with the fuzzified properties. The fuzzy arithmetic steps are implemented for the composite property evaluation via the triangular membership functions (α-cut method) and solved numerically via finite element techniques. The variational technique and the classical Hamilton's principle are utilized to derive the necessary structural governing equation of the laminated structure including the fuzzified properties. Subsequently, the frequency and the static deflection values are obtained computationally via an efficient home-made computer code (MATLAB environment) with the help of current higher-order finite element model. Further, different kinds of numerical examples are solved using the current higher-order fuzzified model to demonstrate the convergence and comparison behaviour. The differences between the available deterministic (3D-FEM and exact solution) and the uncertain (fuzzified) structural responses indicate the necessity of the current model including the accuracy level. Lastly, the influence of the individual and the combined effect of the material (fuzzified elastic property) and the geometrical parameters on the final structural responses are computed and inferred in details.

1. Introduction

From last few decades, the application of the advanced composite materials has increased exponentially all over the globe, majorly in the weight sensitive industries (aerospace, marine, aircraft, modern civil constructions, automobile, etc.) because of their excellent tailor-made properties (physical, electrical and mechanical) [1]. The composite structural component exposed to the combined static and the dynamic loading during their operational life. In addition, the structure experience vibration and/or bending leads to failure hence it is important to develop the complete knowledge related the material behaviour including the influence of loading on the final performance. In this regard, a comprehensive volume of

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the research has already been reported using various solution techniques (numerical, analytical and exact method) including the deformation behaviour via different kinematic theories. Most of the analysis described the input parameter using the deterministic type of input parameter, i.e., geometry, material and loading conditions. However, the real-world engineering structures are difficult to define in prices term, the uncertainty always associated with them inherently due to the uncertainty involved in the manufacturing process, application of the loading and uncertain geometry, etc. Therefore, the structural analysis using the deterministic approach as well as using precise input parameter may lead to the significant error in final responses. Hence, it is highly important to include the uncertain parameter for the evaluation of the structural responses instead of the deterministic parameter. In the recent past, a considerable attempt has been made to include the effect of uncertainty either in the geometry, elastic property or loading intensity for the computation of the structural responses (static and dynamic) of the layered composite structure. Further, to establish the current objective few important contributions, i.e., the combination of the recent and the past literature are discussed in the following lines to point out the research gap.

Monte Carlo Simulation (MCS) is one of the oldest approaches and is utilised to compute the uncertainty propagation in the engineering system. Shinozuka and Jan employed MCS [2] for the digital simulation of the multivariate–multidimensional random process as a series of the cosine functions and implemented for the structural dynamic problem [3–5]. Similarly, the deformation and stress behaviour, as well as the influence of uncertainty in the composite structure, is also investigated via the mathematical model in conjunction with MCS [6–10]. Further, a combined model using MCS in conjunction with different method such as stochastic finite element methods [11], statistical finite element methods [12] and Preconditioned Conjugate Gradient (PCG) method [13] is used to obtain the structural responses. Though, MCS is expensive in terms of calculation time [14] still preferred for the uncertain analysis due to its solution accuracy and robustness. Due to the higher solution time, the researcher prefers to compute the structural responses of the layered composite using other available first-order perturbation techniques parallel to MCS including the random elastic properties [15–21]. Dynamic behaviour of composite structures is also evaluated using different shear deformable, differential quadrature and finite element (FE) method [22–27]. Subsequently, new expansion techniques (orthogonal polynomial expansion method, Neumann and polynomial chaos expansions and Karhunen–Loeve expansion method) [28–31] are adopted for the modelling of the composite structure including either the single or the multiple random sources.

After the advancement of the soft computing techniques and successful implementation in structural analysis, the hybrid modelling of fuzzy set theory in conjunction with finite element method (FFEM) has also been employed for the complex structural problem. The FFEM model has been implemented to investigate the static and dynamic responses of different engineering structures [32–37]. Similarly, the dynamic responses of the laminated structure and the smart structures are computed using the FFEM [38,39]. In addition, the FFEM model has been implemented to investigate the frequency and deflection responses of the beam structure using the imprecise fuzzified properties [40–42]. The analytical solutions of the thin composite beam structure is reported [43] using the fuzzy membership function approach and highlighted the influence of material uncertainties on structural responses. Additionally, the multi-source uncertainty [44,45] including the hybrid reliability analysis [46] of the structure and structural components are studied by using a non-probabilistic reliability-based topology optimization technique.

The inclusive review indicated that the implementation of the FFEM technique for the analysis of the layered structure is not studied extensively. Also, the review indicated that major study related to the 1D or beam-like structures. Moreover, the modelling of the layered structure via higher-order kinematics and computation of the related structural responses (frequency and deflection) using the FFEM technique and fuzzified properties have not reported in the past. However, the structural modelling including the subsequent FEM solutions of the responses (static, dynamic and buckling) reported in large numbers using the random elastic properties via perturbation techniques and MCS. Hence, the current article first-time developed a novel FE model for the layered composite structure in the framework of the higher-order kinematic theory including the fuzzified elastic constants through the fuzzy arithmetic steps and α-cut membership functions. The final governing equations for the static and the modal analysis are derived using the variational technique. The final solutions have been obtained computationally via an own MATLAB code using the current formulations. Finally, the influences of the geometrical and material parameter including the fuzzified elastic property have been computed via the different numerical examples and the inferences in detail.

2. Theoretical formulation

2.1. Concepts of fuzzy arithmetic operations

The fuzzy set theory has the potential for the random modelling of the input parameter of the layered structure. Before the implementation, the fundamental concept of fuzzy set theory, the concept of fuzzy numbers, membership function including the fuzzy arithmetic steps are discussed in the following line. Let U be defined as a classical set of object, also named as the *universe* [40]. The generic element of the universe, U is defined as u. Now, the membership in a classical subset A of U can be represented as a characteristic function, μ_A from the classical set of object, U to valuation set $\{0, 1\}$:

$$\mu_A(u) = \begin{Bmatrix} 1, & \text{if } u \in A, \\ 0, & \text{if } u \notin A \end{Bmatrix} \tag{1}$$