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Performance analysis of fuzzy systems based on quintuple implications method

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ABSTRACT

To improve the quality of approximate reasoning in fuzzy system, quintuple implication principle (QIP) to resolve fuzzy modus ponens (FMP) and fuzzy modus tollens (FMT) problems has been proposed by Zhou et al. [32]. The same approximation as Mamdani-type fuzzy inference can be reached by fuzzy reasoning with QIP method for Gödel implication. It therefore is essential to establish some fundamental properties of fuzzy inference system with QIP method. This paper mainly investigates the robustness and universal approximation capability of fuzzy inference system with QIP method. Firstly, we present the QIP solutions of FMP and FMT for R-, S-, QL-, f- and g-implications. And then the robustness of fuzzy inference system with QIP method is discussed. Finally, we study the universal approximation properties of multiple-input and single-output (MISO) fuzzy systems with QIP method for R-, S- and QL-implications. These results reveal that the QIP method possesses better performance in fuzzy rule-based system.

1. Introduction

Fuzzy reasoning has been successfully applied for model-based control, data mining, artificial intelligence, image processing, decision making and so on. Generally speaking, fuzzy reasoning seeks to infer new fuzzy logical propositions from if–then rules, as an extension of classical logical inference. Its fundamental patterns are fuzzy modus ponens (FMP) and fuzzy modus tollens (FMT) derived from modus ponens (MP) and modus tollens (MT) in the classical logic. And they can be intuitively represented as:

FMP	FMT
Premise 1: IF x is A THEN y is B	Premise 1: IF x is A THEN y is B
Premise 2: x is A'	Premise 2: y is B'
Conclusion: y is B',	Conclusion: x is A'.

To obtain an approximation $B'(A')$ of $B(A)$, the compositional rule of inference (CRI) method is proposed by Zadeh in 1973 [31]. In Zadeh's CRI, Premise 1 is translated into a fuzzy relation R through Zadeh implication. $B'(A')$ is then calculated by combining $A'(B')$ and fuzzy relation R with the sup-min composition. After, the general CRI method for FMP and FMT are developed as follows:

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$$B'(y) = \bigvee_{x\in U} A'(x) * (A(x) \to B(y)),$$
$$A'(x) = \bigvee_{y\in V} B'(x) * (A(x) \to B(y)),$$

where $*$ is a t-norm, $\to$ is an implication.

Although CRI method has been successfully applied in fuzzy reasoning, there are some deficiencies in CRI method as pointed out by some researchers [3,8,20,24–26]. In order to provide a logical foundation, Wang proposed triple implication principle (TIP) for fuzzy reasoning [26]. Since it characterizes the solution as the one which is best supported by the rule $A \to B$ and premise, TIP method possesses reductivity property and logical sense. And then, it becomes a favorite topic to analyze fuzzy reasoning with TIP method [16,18,22,23].

However, the TIP method makes the computed solutions for FMP and FMT sometimes useless or misleading [32]. To improve the quality of fuzzy inference, Zhou et al. proposed quintuple implication principle (QIP) for FMP and FMT [32]. And they obtained the QIP solutions of FMP and FMT for some important implications. Most importantly, it is showed that Mamdani-type fuzzy inference is same as fuzzy inference with QIP method using Gödel implication. These surprising results trigger our interest to apply fuzzy reasoning with QIP method in fuzzy system.

It is well known that the results of fuzzy inference completely depend on the choice of fuzzy sets of antecedents and consequences as well as fuzzy connectives linking antecedents and consequences. As one of the most important fuzzy connectives linking antecedents and consequences, fuzzy implication plays an important role in fuzzy reasoning because it is utilized to formalize "if ... then" rule in fuzzy system. There exist many families of fuzzy implications, such as R-, S-, QL-, D-, f- and g-implications and so on. Therefore, it is valuable to investigate the action of these implications on QIP method.

When a control system is sensitive to small deviations of the inputs, ones need to find the maximum tolerance of the system with respect to these perturbations, referred as the system's robustness. Thus, it is very meaningful that provide a mathematical scheme of robustness to resistant against deviation of human expertise from its corresponding mathematically quantitative representations. Robustness of fuzzy reasoning with CRI and TIP methods have been studied by many researchers. For example, Nguyen et al. introduced the robust properties of various fuzzy connectors [21]. They also showed that min($\wedge$) and max($\vee$) are the most robust operators. Ying proposed the concepts of maximum and average perturbations of fuzzy sets [30]. By the notion of δ equalities of fuzzy sets, Cai studied robustness of fuzzy reasoning [4]. Li et al. discussed the robustness of fuzzy logic connectives and fuzzy reasoning [11,12]. Dai et al. discussed perturbation of fuzzy reasoning with Minkowski distances [6]. Dai et al. considered the robustness of TIP method [7]. Wang and Duan studied the robustness of TIP method with respect to finer measurements [27]. However, there are no results on the robustness of fuzzy reasoning with QIP method so far.

At present, there are three types of fuzzy systems which have been widely applied in various fields: Mamdani, Takagi–Sugeno and Boolean fuzzy systems. The major difference among the three classes lies in the choice of fuzzy implications to interpret a fuzzy rule. In general, Mamdani and Takagi–Sugeno fuzzy systems choose t-norms as implications, such as the min($\wedge$) or product ($\times$) operator, while Boolean fuzzy systems utilize genuine multi-valued implications that mainly contain R-, S- and QL-implications. Obviously, Mamdani and Takagi–Sugeno fuzzy systems lack a strict logical foundation from a mathematical point of view.

Since QIP method has the firm logic foundation and contains Mamdani's method as a special case, it is necessary to consider the fuzzy system with QIP method. It is well known that fuzzy system is just particular types of nonlinear functions mapping its input to output. So for fuzzy system used as controller or decision maker or any others, it is interesting to know whether a fuzzy model can always be established which can approximate nonlinear functions to arbitrary accuracy [28]. The questions are both theoretical and practical importance. This begs an interesting question: Are the fuzzy systems with QIP method universal approximators? To the best of our knowledge, there is no answer about this question.

Having these in mind, this paper is organized as follows. In Section 3, the QIP solutions of FMP and FMT for R-, S-, QL, f- and g-implications are given. In Section 4, we discuss the recovery properties of fuzzy inference with QIP methods with R-, S-, QL, f- and g-implications. Section 5 shows the robustness of fuzzy inference systems with QIP methods for some specific fuzzy connectives. In Section 6, the universal approximation properties of MISO fuzzy systems with QIP method are investigated.

2. Preliminary

In this section, we briefly summarize some basic concepts and results that are needed for further treatment.

Definition 2.1. [17] A function $N : [0,1] \to [0,1]$ is called a fuzzy negation if

N1. $N(0) = 1, \ N(1) = 0$;

N2. $N(x) \geq N(y)$ if $x \leq y, \ \forall \ x, y \in [0,1]$.

Further, a fuzzy negation N is strict if it satisfies the following properties:

N3. N is continuous;