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# The capacitated vehicle routing problem with evidential demands ☆

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ABSTRACT

We propose to represent uncertainty on customer demands in the Capacitated Vehicle Routing Problem (CVRP) using the theory of evidence. To tackle this problem, we extend classical stochastic programming modelling approaches. Specifically, we propose two models for this problem. The first model is an extension of the *chance-constrained programming* approach, which imposes certain minimum bounds on the belief and plausibility that the sum of the demands on each route respects the vehicle capacity. The second model extends the *stochastic programming with recourse* approach: for each route, it represents by a belief function the uncertainty on its recourses, *i.e.*, corrective actions performed when the vehicle capacity is exceeded, and defines the cost of a route as its classical cost (without recourse) plus the worst expected cost of its recourses. We solve the proposed models using a metaheuristic algorithm and present experimental results on instances adapted from a well-known CVRP data set.



## 1. Introduction

In the Capacitated Vehicle Routing Problem (CVRP), we are given a fleet of vehicles with identical capacity located at a depot and a set of customers with known demands located on the vertices of a graph. The goal of this problem is to determine a route for each vehicle, such that the set of routes for all the vehicles has the least total cost, all customer demands are fully serviced, the capacity of each vehicle is always respected and each customer is visited by exactly one route. The CVRP is NP-hard since it contains the traveling salesman problem as a particular case (one route and unbounded capacity). It can be written as an integer linear program. The CVRP has generated a large body of research, since it belongs to the class of local transportation or delivery problems affecting the most expensive component in the distribution network [8].

Yet, many industrial applications are confronted with uncertainty on customer demands in their distribution problems involving the CVRP, and the exact customer demands are mostly revealed when the servicing vehicles arrive at the customers. Accordingly, several authors (see, *e.g.*, [28,29] and the references therein) tackled this issue by assuming that customer demands are random variables and the associated problem is the well-known Capacitated Vehicle Routing Problem with Stochastic Demands (CVRPSD). Two of the most widely-used frameworks for modelling stochastic problems, such as the

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CVRPSD, are the Chance-Constrained Programming (CCP) approach and the Stochastic Programming with Recourse (SPR) approach [7]. Modelling the CVRPSD via CCP amounts to using a probabilistic capacity constraint that requires the probability of respecting the capacity constraint to be above a certain threshold. The CCP modelling technique does not consider the additional cost of recourse (or corrective) actions necessary if capacity constraints fail to be satisfied. The SPR approach does consider situations needing recourses and it aims at minimizing the initially-planned travel cost plus the expected cost of the recourses executed along routes, *e.g.*, returning to the depot and unloading in order to bring to feasibility a violated capacity constraint.

The probabilistic approach to modelling uncertainty is not necessarily well-suited to all real-life situations. In particular, its ability to handle epistemic uncertainty (uncertainty arising from lack of knowledge) has been criticised [4,1]. The typical approach to representing basic epistemic uncertainty is the set-valued approach [26]. It is sensible when, *e.g.*, all that is known about the customer demands is that they belong to some intervals. This kind of uncertainty in the CVRP is generally addressed using robust optimisation, where one optimises against the worst-case scenario, that is, one wants to obtain solutions that are robust to all realisations of customer demands that are deemed possible (see, *e.g.*, [49]). However, the set-valued approach to uncertainty representation may be too coarse and may thus lead to solutions that are too conservative, hence not useful.

In the last forty years, the necessity to account for all facets of uncertainty has been recognized and alternative uncertainty frameworks extending both the probabilistic and set-valued ones have appeared [4]. In particular, the theory of evidence introduced by Shafer [47], based on some previous work from Dempster [15], has emerged as a theory offering a compromise between expressivity and complexity, which seems interesting in practice as its successful application in several domains testifies (see [18] for a recent survey of evidence theory applications). This theory, also known as belief function theory, may be used to model various forms of information, such as expert judgements and statistical evidence, and it also offers tools to combine and propagate uncertainty [1].

In the context of the CVRP, the theory of evidence may be used to represent uncertainty on customer demands leading to an optimisation problem, which will be referred to as the CVRP with Evidential Demands (CVRPED). Using the theory of evidence in this problem seems particularly interesting as it allows one to account for imperfect knowledge about customer demands, such as knowing that each customer demand belongs to one or more sets with a given probability allocated to each set – an intermediary situation between probabilistic and set-valued knowledge. In this paper, we propose to address the CVRPED by extending the CCP and SPR modelling approaches into the formalism of evidence theory. Although the focus will be to extend stochastic programming approaches, we will also connect our formulations with robust optimisation.

To our knowledge, evidence theory has not yet been considered to model uncertainty in large-scale instances of an NP-hard optimisation problem like the CVRP. Indeed, it seems that so far, only other non-classical uncertainty theories, and in particular fuzzy set theory [50,9,42,11], have been used in such problems. Besides, modelling uncertainty in optimisation problems using evidence theory has concerned only continuous design optimisation problems[1] [41,48] and continuous linear programs [40]. Specifically in [41], the reliability of the system is optimized, while uncertainty is handled by limiting the plausibility of constraints violation into a small degree; while in [48] the problem was handled differently, and the plausibility of a constraint failure was converted into a second objective to the problem that should be minimized. Of particular interest is the work of Masri and Ben Abdelaziz [40], who extended the CCP and SPR modelling approaches, in order to model continuous linear programs embedding belief functions, which they called the Belief Constrained Programming (BCP) and the recourse approaches, respectively. In comparison, in this work, we generalise CCP and SPR to an *integer* linear program involving uncertainty modelled by evidence theory. Borrowing from [40], we propose to model the CVRPED by methods that may be called the BCP modelling of the CVRPED and the recourse modelling of the CVRPED. For both models, the resolution algorithm is a simulated annealing algorithm; we use a metaheuristic, as the CVRPED derives from the CVRP, which is NP-hard.

The paper is structured as follows. Section 2 summarises the basic preliminaries on the CVRP and on the CVRPSD modelling via CCP and SPR, along with the necessary background on evidence theory. In Section 3, the BCP model and the recourse model for the CVRPED are presented and some of their properties are studied. In Section 4 we solve the BCP model and the recourse model of the CVRPED using a simulated annealing algorithm and perform experiments on instances generated from CVRP benchmarks. In Section 5, we conclude and state the perspectives of the present work.

## 2. Background

This section recalls necessary background on the CVRP, the CVRPSD and its stochastic programming formulations, as well as some concepts of belief function theory needed in this paper.

[1] Designing physical systems in the engineering field using optimisation techniques, so design costs are minimized, while the system performance is fulfilled [2].