



Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar
An analogical hexagon[☆]

Jean-Yves Beziau

Federal University of Rio de Janeiro, Brazil



ARTICLE INFO

Article history:

Received 24 August 2016

Received in revised form 12 December 2017

Accepted 12 December 2017

Available online xxxx

Keywords:

Analogy

Opposition

Trichotomy

ABSTRACT

In a first part we discuss the different ways to go beyond dichotomies, using trichotomies and hexagons of opposition. In a second part we show how to produce a hexagon with analogy. In a third part we investigate the meaning given to analogy and related notions with this hexagon presenting some examples.

© 2017 Elsevier Inc. All rights reserved.

Introduction

Analogy is a very famous and popular notion. Everybody likes to make some analogies. Roughly speaking, making an analogy is to compare two *different* things, stressing one *similar* feature, which is *transposed* from one thing to another one, shedding a new light on it. Considering this transportation, we can consider that analogies are metaphors (cf. the etymology of “metaphor”).¹ Making analogies is an art, the result can be a *chef d'oeuvre* or an ugly and ridiculous thing when the mayonnaise has not succeeded.

Analogy is most of the time a bit challenging if not controversial, even when performed by a good writer like Leo Tolstoy: “Truth, like gold, is to be obtained not by its growth, but by washing away from it all that is not gold”, or a virtuosic mathematician like Stefan Banach: “A mathematician is a person who can find analogies between theorems; a better mathematician is one who can see analogies between proofs and the best mathematician can notice analogies between theories. One can imagine that the ultimate mathematician is one who can see analogies between analogies”.²

Analogy looks in some way as opposed to rationality. But an analogy is not necessarily completely irrational. “Ana-logon” is not “a-logon”. “Ana” means above. Although Plato’s cave can be seen as an analogy, there is in ancient Greece a much more precise view of analogy, this is *proportional analogy*: *A* is to *B* as *C* is to *D*. H. Prade and G. Richard have recently provided a detailed logical theory of it. For that they used the theory of opposition, in particular the square and the hexagon of opposition (see [42] and also [43]).

In the present paper we are also using the theory of opposition, but we consider that though analogical proportion is part of analogy, it does not reduce to it. We present a hexagon of opposition figuring analogy in a wider sense. This hexagon is not constructed by breaking the dichotomy identity/difference into a trichotomy but by inserting it in a framework with two more dichotomies and a trichotomy involving opposition itself.

[☆] This paper is part of the virtual special issue on tri-partition, edited by Davide Ciucci and Yiyu Yao.

E-mail address: jyb@ufrj.br.

¹ Analogies can be considered as a special case of metaphors, not all metaphors are necessarily analogies.

² This is a rather metamathematical analogy, the Bourbachi mathematician André Weil wrote a letter to his sister about a true mathematical analogy, see [47]; see also the work of Polya [40,41] and the paper by Pimm [39].

In a first part we discuss the different ways to go beyond dichotomies, using trichotomies and hexagons of opposition. We explain the theory and give some examples. In a second part we show how to produce a hexagon with analogy. In a third we investigate the meaning given to analogy with this hexagon, discussing the related notions which appear in the hexagon, in particular similarity, and presenting some examples.

In this paper, as in our recent paper “Possibility, imagination and conception” [12], our methodology has three aspects: structurality, equilibrium between descriptivity/normativity, prototypical examples. Let us explain better this methodology in the context of analogy. Analogy manifests first of all as a word: “analogy”. It is not only a word, but on the other hand we don’t believe in an inner nature of analogy. The word “analogy” and the notion corresponding to it have many different significations. Our objective here is not to describe all these significations. Our approach is normative, we want to fix the meaning of this notion, not in a purely arbitrary way, but taking reasonably in account how it is generally understood, and also relating this notion with a small family of close-by notions. In other words: we are developing a *theory*, a *structuralist* theory. For doing that we are using the theory of opposition.³ This paper is self-contained and can be considered as an introduction to the theory of opposition, through a particular example.

The theory of opposition is a quite simple theory that is based on some logical notions and on some diagrams. It can be considered as part of universal logic whose idea is not to construct a particular system, but to develop concepts, tools and frameworks that can be useful for the comparison, improving and building of logical systems (see [2]). It is *foundational* in this sense. It can be seen as a first step for further developments or as giving a better understanding of already specific systematization. This is how we can see the relation between our hexagon of analogy and the theory of proportional analogy of Prade and Richard.

The aim of our paper is not to be precise in the sense of being specific. Our definition of analogy is *general* in the sense that it leaves space for many possible interpretations. This is in fact the spirit of modern abstract mathematics which went up to the stage of axiomatic emptiness, in particular with the work of Birkhoff in Universal Algebra (for a discussion about this, see [6]). But our definition is not imprecise in the sense of confusion. To give a position to the notion of analogy in a hexagon of opposition is rigorous and rational: on the one hand because the theory of opposition is itself a logico-mathematical theory, on the other hand because we are performing reasoning to apply this theory to the notion of analogy. We are not presenting this hexagon as an *axiom* for the theory of analogy in the original sense of the word, i.e. as something obvious, nor just as an hypothesis. We are trying to explain why and how this hexagon of analogy makes sense.

For doing that we are also using images and prototypical examples. This is entirely part of our methodology. Images are still far to be accepted in philosophy, the rejection can be traced back to Ancient Greek philosophy, petrified in Plato’s cave, which paradoxically is an image for the rejection of images. But it is not because images are often badly used or misleading that they cannot be used in an intelligent way. It is much more common to use images in science, they are used in a rather precise way, through some diagrams, like indeed in the theory of opposition.

According to the famous proverb: *A picture is worth a thousand words*. Many times an image gives an approximation that can be then developed in a more sophisticated way. Depending on the circumstances, we need, or we need not, to go to a deeper/higher level. If we want to go to the baker, we don’t necessarily need to know the law of physics explaining the move of our body. To use the right level of sophistication is an art.⁴

In this paper we are furthermore using images through the methodology of prototypical examples. The idea of prototypical examples is relatively independent of imaging. The idea is to *think* of the general through the particular, finding a typical example.⁵ Giving an image reinforces the power of the exemplification, in the sense that we can directly see what it is. This process has been using in mathematics in the case of the sign for equality/identity: “=”. This sign is an image introduced by Robert Recorde ([44]) in the 16th century and which has been used informally during many centuries and is still widely used informally. Few books of mathematics defined what it is, as if it was obviously clear. The theory of identity has started to be developed by logicians at the 20th century but is still an open theory (see e.g. [4]). It is not because people are using a symbol like “=” that they are doing a “formal” theory. It can be a first step in this direction, or not – in astrology people are also using symbols, e.g. ζ (For a discussion about that see our paper [5].)

We will come back in more details on the question of images and prototypes at the beginning of section 3 where we are using this technology.

1. From dichotomies to hexagons

1.1. Why breaking the dichotomy?

Pythagoras’ table of opposites (Fig. 1) was an interesting departure point for the systematization of thought.

³ This is not the only possibility, an other interesting structuralist approach to analogy was developed by S. Klein, see [34].

⁴ One of the referees suggested to present an interesting formal model of our hexagon based on vectors. We will not present it here, because on the one hand it is his own idea that he is welcome to develop in a forthcoming paper, on the other hand this is only one possible models among others of our hexagon. Regarding other more informal models of analogy based on words or images, here are two useful references kindly provided by the same referee: [45], [33]. Among previous works on analogy it is also worth citing here the paper by Grize, see [31].

⁵ It is possible to develop a logical theory justifying such methodology, this has been done in particular by J.-P. Desclés (see e.g. [29]).

Download English Version:

<https://daneshyari.com/en/article/6858817>

Download Persian Version:

<https://daneshyari.com/article/6858817>

[Daneshyari.com](https://daneshyari.com)